Introductory Remarks

Independent claim 40 stands finally rejected under 35 U.S.C. § 102(b) as being anticipated by United States Patent no. 5,581,616 entitled "Method and Apparatus for Digital Signature Verification" that issued December 3, 1996, on a patent application filed by Richard E. Crandall ("the Crandall '616 patent").

Applicant respectfully submits that regarding the final rejection of independent claim 40, Exhibits A through C hereto irrefutably establish that the rejection of claim 40 resides either upon:

1. an incorrect or fallacious reading of the text of the Crandall '616 patent; or
2. a deliberate, conscious avoidance of the Crandall '616 patent's text.

Applicant acknowledges that the "**public source 813**" disclosed in the cited reference receives, either expressly or implicitly:

1. **ourPub** from a sender; and
2. **theirPub** from a receiver.

In finally rejecting independent claim 40 the March 29, 2005, Office Action on page 4 beginning in line 5, without providing any justification therefor, summarily declares:

> it is **implicit** in the disclosure that **the plurality of public quantities is stored by the sender**. (Emphasis supplied.)

First, essentially without reference to the text of the Crandall '616 patent, Applicant respectfully suggests that it would be equally logical to summarily declare:

> it is **implicit** in the disclosure that **the plurality of public quantities is stored by the receiver (recipient)**.

It is clear that both of the preceding, conflicting summary declarations can't be true simultaneously. **Either one or the other of the preceding summary declarations must be false**.

Now, perhaps going outside the text of the Crandall '616 patent, Applicant respectfully further suggests that it also would be equally logical to summarily declare:

> it is **implicit** in the disclosure that **the plurality of public quantities is stored by a trusted third party**.

It is clear that all three of the preceding summary declarations can't be simultaneously true. **Only one (1) of the three (3) preceding summary declarations can be true**.

Now turning to the Crandall '616 patent, in column 7 at lines 23 through 32 the text expressly states "[n]ext, **parameters are established for both sender and recipient**,"[1] i.e. the parameters:

1. a "bit-depth" q which is a fast class number;
2. k, so that F_{p^k} will be the field;
3. The initial x-coordinate $(x_1, y_1) \in F_{p^k}$; and

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[1] Note that the Crandall '616 patent's text doesn't state that the parameters are established **by** both sender and recipient, if that were somehow technologically possible.

4. The curve-defining parameter a ϵ F_{pk}.

Clearly the preceding text excerpted from column 7 of the Crandall '616 patent:

1. **irrefutably contradicts the** March 29, 2004, Office Action's **summary declaration that implicitly the sender stores the plurality of public quantities**; and
2. **proves that the parameters are established by neither the sender nor by the recipient**, but rather are established for the sender and recipient.

Furthermore, the text of the Crandall '616 patent in column 16, lines 1 through 9, criticizes the RSA cryptosystem because a "user cannot generate its own **private key** in the RSA system." Contrasting the Crandall '616 patent's elliptic curve cryptosystem with the RSA cryptosystem, the Crandall '616 patent in column 16 states:

> [t]he present invention does not require that the private key be a prime number. Therefore, users can generate their own private keys, **so long as a public key is generated and published using correct and publicly available parameters p, F_{pk}, (X_1/Z) and "a"**.

Thus, the preceding text from column 16 discloses that:

1. there exists cryptosystems which are so mathematically difficult that a "user" cannot generate their own private key, no less their own public key;

2. for such cryptosystems, a trusted third party must establish both the private and public keys; and

3. announces as a significant advance in cryptosystem technology a user's ability to select their own private key.

If a user's ability to select their own private key were such a significant advance in cryptosystem technology that the Crandall '616 patent specifically mentions it, wouldn't the Crandall '616 patent be reasonably expected to similarly expressly announce in the reference's text a user's ability to establish the elliptic curve cryptosystem parameters p, F_{pk}, (X_1/Z) and "a." The only reasonable inference which can be drawn from the Crandall '616 patent's failure to specifically describe user's ability to establish the parameters p, F_{pk}, (X_1/Z) and "a" is that establishing those parameters is generally beyond a user's capability due to their mathematical complexity and difficulty of the esoteric elliptic curve cryptosystem. Consequently, the text excerpted above from column 16 confirms the statement excerpted from column 7 that the "**parameters are established for both sender and recipient**," probably by a highly mathematically-skilled, trusted third party.

Clearly the equally likely possibilities that the parameters are established by the sender, by the receiver or by a trusted

third party outlined above demonstrates the fallacy of the March 29, 2005, Office Action's summary declaration on page 4 that:

> it is **implicit** in the disclosure that **the plurality of public quantities is stored by the sender**. (Emphasis supplied.)

Confirming the preceding logical proof that the March 29, 2005, Office Action's summary declaration is false, the text excerpted above from column 7 and from column 16 of the Crandall '616 patent irrefutably prove that the March 29, 2005, Office Action's summary declaration is fraudulent.

Independent claims 1, 14 and 27 stand finally rejected under 35 U.S.C. § 103(a) as being unpatentable over:

a. United States Patent no. 4,200,770 entitled "Cryptographic Apparatus and Method" that issued April 28, 1980, on a patent application filed by Martin E. Hellman, Bailey W. Diffie and Ralph C. Merkle ("the Hellman, et al. patent"); in view of

b. "Applied Cryptography" © 1996 by Bruce Schneier, published by John Wiley & Sons, Inc. ("Schneier").

This response demonstrates how **applying the Hellman, et al. patent in view of Schneier** to element "a. ports" in the body of independent claim 27 with:

1. **claim's 27 receiver being the converser 12** depicted in FIG. 12 of the Hellman, et al. patent; and

2. **claim's 27 sender being the converser 11** depicted in FIG. 12 of the Hellman, et al. patent;

fails to render obvious independent claim 27. Stated most simply, in comparison with the text of independent claim 27 the Hellman, et al. patent lacks sufficient quantities, either public or exchanged between a sending and receiving cryptographic unit, to render independent claim 27 obvious.

AMENDMENTS

There are no **Amendments to the Specification**.

There are no **Amendments to the Claims**.

There are no **Amendments to the Drawings**.

Remarks/Arguments begin on page 9 of this Response.

REMARKS

In view of the following remarks, Applicant respectfully requests reconsideration of the present application.

Objections and Rejections

The Office Action dated March 29, 2005:

1. finally rejects claims 40 and 41 under 35 U.S.C. § 102(b) as being anticipated by the Crandall '616 patent;
2. finally rejects claims 1-5, 12-18, 25-31, 38 and 39 under 35 U.S.C. § 103(a) as being unpatentable over:
   a. the Hellman, et al. patent; in view of
   b. Schneier; and
3. rejects claims 6-11, 19-24 and 32-37 under 35 U.S.C. § 103(a) as being unpatentable over:
   a. the Hellman, et al. patent; in view of
   b. Schneier; and further in view of
   c. United States Patent no. 5,159,632 entitled "Method and Apparatus for Public Key Exchange in a Cryptographic System" that issued October 27, 1992, on an application filed by Richard E. Crandall ("the Crandall '632 patent").

The Claimed Invention

The invention, as embodied in **independent claim 40**, is a three (3) step method by which a receiving unit R that receives a message M and a digital signature authenticates the digital signature.

1. Step 1 requires **retrieving a plurality of public quantities from a publicly accessible repository**.
2. Step 2 requires **evaluating expressions of at least two (2) different verification relationships** using:
   a. **the received digital signature**; and
   b. **the plurality of public quantities**;
3. Step 3 requires **comparing pairs of results obtained by evaluating the expressions of the at least two (2) different verification relationships**.

The invention, as embodied in **independent claim 27**, is **a cryptographic unit** that includes:

1. **ports**; and
2. **a cryptographic device**.

When **the cryptographic unit** encompassed by independent claim 27 **is to receive a cyphertext message M**, the **ports**:

1. **store a plurality of public quantities in a publicly accessible repository**; and

2. **receive** via the communication channel I **a plurality of sender's quantities from a sending cryptographic unit**.

The receiving cryptographic unit uses the plurality of sender's quantities and at least some of the plurality of public quantities in **computing**:

1. **at least one receiver's quantity** which said receiving cryptographic unit transmits via the communication channel I to said sending cryptographic unit; and
2. a cryptographic key K.

When **the cryptographic unit** encompassed by independent claim 27 is **to send the cyphertext message M**, the **ports retrieve the plurality of public quantities from the publicly accessible repository**. The **sending cryptographic unit** then uses the retrieved plurality of public quantities in **computing**:

1. **the plurality of sender's quantities** which the sending cryptographic unit transmits via the communication channel I to the receiving cryptographic unit; and
2. after receiving via the communication channel I the receiver's quantity from the receiving cryptographic unit, **the cryptographic key K**.

In addition to the ports, **the cryptographic unit** encompassed by independent claim 27 also includes **a cryptographic device**. **The cryptographic device** includes:

1. **a key input port**;

2. **a plaintext port**; and

3. **a cyphertext port**.

The key input port receives the cryptographic key K. The plaintext port:

1. accepts a plaintext message P for encryption into the cyphertext message M; and

2. delivers a plaintext message P obtained by decrypting a received cyphertext message M.

The cyphertext port, which is coupled to one a transceiver included in a cryptographic system:

1. transmits the cyphertext message M to the transceiver; and

2. receives the cyphertext message M from the transceiver.

The Cited References

The Crandall '616 Patent

That portion of the January 28, 2005, response to the August 24, 2005, Office Action which describes the disclosure of the Crandall '616 patent is hereby incorporated by reference as though fully set forth here.

Furthermore, Applicant apologizes for the bulk of the following further description of the disclosure of the Crandall

'616 patent. However, the March 29, 2005, Office Action's unjustified summary declaration that the Crandall '616 patent implicitly discloses that the sender stores the plurality of public elliptic curve cryptosystem parameters compels Applicant to prove a negative proposition, i.e. that the Crandall '616 patent **does not** implicitly discloses that the sender stores the plurality of public elliptic curve cryptosystem parameters. While proof of a positive proposition is relatively easy, i.e. find a single example or instance of the proposition's truth, proving a negative proposition requires exhaustively analyzing every detail of the Crandall '616 patent to disprove the March 29, 2005, Office Action's summary declaration.

Exhibit A hereto lists by column and line number in the Crandall '616 patents all instances of the words or phrases:

1. public;
2. publicly;
3. separate source 813;
4. public source 813;
5. source 813;
6. ourPub;
7. myPub; and
8. theirPub.

Exhibit B hereto presents the entire text of the Crandall '616 patent hi-lited to indicate every occurrence of:

1. either of the words "public" or "publicly;"
2. the phrases "separate source 813," "public source 813" and "source 813;"
3. the words "ourPub," "myPub" and "theirPub."

Exhibit C hereto excerpts all texts from the Crandall '616 patent which report storing data into a publicly accessible repository:

1. for purposes of authentication, i.e. for digital signature use; and
2. for any use in connection with the elliptic curve encryption schemed disclosed in the Crandall '616 patent.

Applicant respectfully submits that, as irrefutably established by Exhibits A-C hereto and for the reasons set forth below, the Crandall '616 patent expressly or even implicitly describes computation and publication:

1. by a sender of only **ourPub** using the sender's ourPri;[2] and

2. by a receiver of only **theirPub** using the receiver's theirPri.[3]

Koblitz Authentication

Specifically, in column 5 at lines 42 through 50 Exhibit C's excerpt from the Crandall '616 patent describes data stored into a publicly accessible repository for an authentication scheme described in "A Course in Number Theory and Cryptography" (Koblitz, 1987, Springer-Verlag, New York). This text from the Crandall '616 patent discloses that:

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[2] The Crandall '616 patent in column 3 at lines 10-11 states initially that "a **sender** has a public key, referred to as 'ourPub'." The Crandall '616 patent regarding "Elliptic Curve Public Key Exchange" in column 7 at lines 62-64 definitively states.

> In the following description, the terms "**our**" and "**our end**" refer to the **sender**. The terms "**their**" and "**their end**" refer to the **receiver**. (Emphasis supplied.)

[3] The Crandall '616 patent in column 3 at lines 11-12 states initially that "a **receiver** has a public key, referred to here as 'theirPub'." The Crandall '616 patent regarding "Elliptic Curve Public Key Exchange" in column 7 at lines 62-64 definitively states.

> In the following description, the terms "**our**" and "**our end**" refer to the **sender**. The terms "**their**" and "**their end**" refer to the **receiver**. (Emphasis supplied.)

1. a user A, i.e. the sender, has a public key f_A;
2. a user B, i.e. the receiver, has a public key f_B; and
3. user B, who receives an ciphertext from user A, uses both the receiver's public key f_B and the sender's public key f_A in decripting a digital signature $f_B f_A^{-1}(P)$ which accompanies the ciphertext.

This excerpt from the Crandall '616 patent does not disclose nor does it suggest:

1. whether the sender's public key f_A is selected by:
   a. the sender; or
   b. an unidentified trusted third party;
1. whether the receiver's public key f_B is selected by:
   a. the receiver; or
   b. an unidentified trusted third party; or
2. how the keys f_A and f_B become publicly known.

ElGamal Authentication

Specifically, in column 5 at lines 53 through 65 Exhibit C's excerpt from the Crandall '616 patent describes data stored into a publicly accessible repository for an ElGamal discrete logarithm scheme using elliptic algebra. This text from the Crandall '616 patent discloses that there exists:

1. a public key ourPub generated with a function of a private key ourPri;[4] together with
2. other ambiguously mentioned data:
   a. a random integer m of approximately q bits;
   b. $X_1/1$ (or perhaps only X_1);
   c. a message digest function M;
   d. myPub, which it appears is likely to be the same as ourPub the sender's public key.[5]

This excerpt from the Crandall '616 patent does not disclose nor does it suggest:

1. whether ourPub and/or myPub are selected by:
   a. the sender; or
   b. an unidentified trusted third party;
2. whether the random integer m, $X_1/1$ (or perhaps only X_1) and/or the message digest function M, are selected by:
   a. the sender;
   b. the receiver; or
   c. an unidentified trusted third party; or

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[4] Which based upon usage elsewhere throughout the Crandall '616 patent should be the sender's public key. However, this description of ElGamal authentication subsequently confusingly uses the term "myPub" which also appears to be a sender's public key.

[5] See the Crandall '616 patent in column 3 at lines 19-22.

3. how ourPub and/or myPub, the random integer m, $X_1/1$ (or perhaps only X_1) and/or the message digest function M become publicly known.

Elliptic Curve Algebra

Specifically, in column 7 at lines 14 through 32 Exhibit C's excerpt from the Crandall '616 patent describes data stored into a publicly accessible repository for use in elliptic curve algebra. This text from the Crandall '616 patent describes establishing the following publicly known parameters.

1. A "bit-depth" q which is a fast class number. The bit-depth q is established so that $p = 2^q - C$.[6]
2. k, so that F_{pk} will be the field.
3. The initial x-coordinate $(x_1, y_1) \in F_{pk}$[7]
4. The curve-defining parameter $a \in F_{pk}$

This excerpt from the Crandall '616 patent, while expressly stating that the "parameters are established **for both sender and recipient**,"[8] does not expressly disclose nor does it suggest:

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6 It appears that this excerpt from the Crandall '616 patent fails to define or describe "C."

7 Note that this appears to be in fact both the initial x-coordinate and the initial y-coordinate.

8 Col. 7, lines 23-24.

1. whether the "bit-depth" q, the field determining parameter k, (x_1, y_1), and/or "a" are "established" by:
   a. the sender;
   b. the receiver; or
   c. an unidentified trusted third party; or
2. how the "bit-depth" q, the field determining parameter k, (x_1, y_1), and/or "a" become publicly known.

Note that the use of the word "for" in the phrase "parameters are established **for** both sender and recipient" declares that the sender and recipient do not establish the "bit-depth" q, the field determining parameter k, (x_1, y_1), and/or "a." Thus, based upon the text of the Crandall '616 patent it appears most likely that the "bit-depth" q, the field determining parameter k, (x_1, y_1), and/or "a" are established by an unidentified trusted third party.

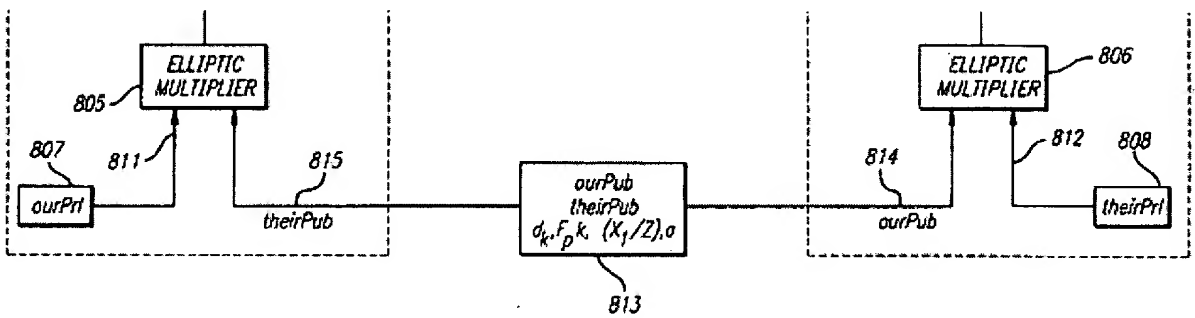
Elliptic Curve Public Key Exchange

Specifically, beginning in column 7 at line 57 and continuing through column 8 at line 14 Exhibit C's excerpt from the Crandall '616 patent describes only **computation and publication** of:

1. **ourPub** by a sender using the sender's ourPri; and
2. **theirPub** by a receiver using the receiver's theirPri.

Disclosure Concerning the "Separate Source 813" Depicted in FIG. 8

The following excerpt from FIG. 8 clearly establishes that the illustration fails to graphically depict storage of any data into the separate source 813. Stated in a different way, FIG. 8 lacks any arrows pointing inward toward the separate source 813.



Specifically, in column 12 beginning at line 63 and continuing to column 13 at line 28 Exhibit C's excerpt from the Crandall '616 patent describes the separate source 813 depicted in FIG. 8. This text from the Crandall '616 patent states that the "separate source 813 stores publicly known information, such as the public keys "ourPub" and "theirPub" of sender 801 and receiver 802, the initial point (x_1, y_1), the field F_{pk}, and curve parameter 'a'." This excerpt from the Crandall '616 patent does not expressly disclose nor does it suggest:

1. that "the initial point (x_1, y_1), the field F_{pk}, and/or curve parameter 'a'" are stored into the separate source 813 by:
   a. the sender;
   b. the receiver; or
   c. an unidentified trusted third party; or
2. how "the initial point (x_1, y_1), the field F_{pk}, and/or curve parameter 'a'" become publicly known.

Fast Class Numbers
<u>Inversionless Parameterization</u>

Specifically, in column 14, lines 16 through 38 Exhibit C's excerpt from the Crandall '616 patent for fast class numbers and inversionless parameterization describes **<u>computation and publication</u>** of:

1. **<u>ourPub</u>** by a sender using the sender's ourPri; and
2. **<u>theirPub</u>** by a receiver using the receiver's theirPri.

Thus, this excerpt from the Crandall '616 patent adds nothing to the reference's disclosure which begins in column 7 at line 57 and continues through column 8 at line 14.

FFT Multiplication

Specifically, in column 15, lines 7 through 32 Exhibit C's excerpt from the Crandall '616 patent describes, with respect to the flow diagram of FIG. 9 how FFT multiplication may be used advantageously to eliminate calculating an inversion. This text from the Crandall '616 patent states that the following publicly known parameters are established for using FFT multiplication.

1. A fast class number $p = 2^q - C$ where q is the bit depth of the encryption scheme
2. k, so that F_{pk} will be the field.
3. An initial point (X_1/Z) on the elliptic curve
4. The curve-defining parameter "a"
5. The sender's public key ourPub = (XZ^{-1})(mod p) where X_1/Z = ourPri$\circ(X_1/1)$
6. The receiver's public key theirPub = (XZ^{-1})(mod p)

First, it is important to observe that this excerpt from the Crandall '616 patent essentially repeats previously analyzed disclosures appearing in:

1. column 7 at lines 14 through 32; and
2. column 7 at line 57 and continuing through column 8 at line 14.

Second, **since the flow diagram of FIG. 9 encompasses generating both the sender's public key ourPub and the receiver's public key**

theirPub, the flow diagram of FIG. 9 cannot disclose a process that is performed exclusively either:

1. by the sender; or

2. by the recipient.

Thus, this excerpt from the Crandall '616 patent does not expressly disclose nor does it suggest:

1. whether the fast class number p, the field determining parameter k, the initial point (X_1/Z) on the elliptic curve, and/or the curve-defining parameter "a" are established by:
    a. the sender;
    b. the receiver; or
    c. an unidentified trusted third party; or
2. how the fast class number p, the field determining parameter k, the initial point (X_1/Z) on the elliptic curve, and/or the curve-defining parameter "a" become publicly known.

FEE Security

Specifically, in column 16, lines 1 through 9 Exhibit C's excerpt from the Crandall '616 patent describes as **a disadvantage that a user cannot generate its own private key in prior crypto-systems such as the RSA system**. This excerpt from the Crandall

'616 patent also describes an advantage provided by the crypto-system described there as follows.

> The present invention does not require that the private key be a prime number. Therefore, **users can generate their own private keys, so long as a public key is generated and published using correct and publicly available parameters p, F_{pk}, (X_1/Z) and "a"**. Col. 16, lines 4-8 (Emphasis supplied.)

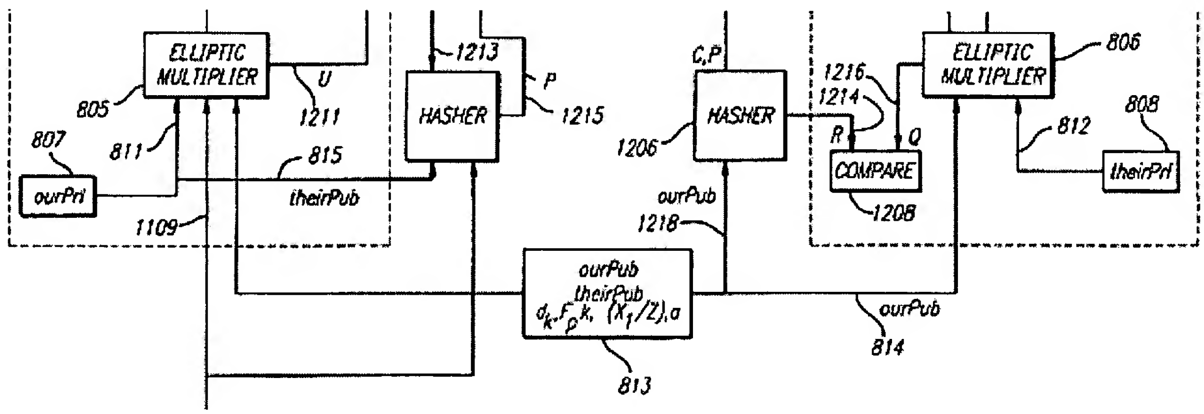
Since this text in the Crandall '616 patent expressly states that users, i.e. senders and recipients, can select their own private keys and then generate their public key from their private key if they use **"correct and publicly available parameters p, F_{pk}, (X_1/Z) and 'a'," this text from the Crandall '616 patent clearly teaches that users, i.e. senders and recipients, do not establish and publish the publicly available parameters p, F_{pk}, (X_1/Z) and "a"**.

Elliptic Curve Cryptosystem Digital Signature Authentication

Specifically, in column 16, line 48 through column 17, line 28 Exhibit C's excerpt from the Crandall '616 patent describes only using publicly available parameters and data for authenticating a digital signature.

Disclosure Concerning the "Separate Source 813" Depicted in FIG. 12

The following excerpt from FIG. 12 clearly establishes that the illustration fails to graphically depict storage of any data into the separate source 813. Stated in a different way, FIG. 12 lacks any arrows pointing inward toward the separate source 813.



Specifically, in column 19 beginning at line 54 and continuing to column 20 at line 37 Exhibit C's excerpt from the Crandall '616 patent describes the separate source 813 depicted in FIG. 12. This text from the Crandall '616 patent essentially repeats previously analyzed disclosures appearing in:

1. column 12 beginning at line 63 and continuing to column 13 at line 28; and
2. column 16, line 48 through column 17, line 28.

This excerpt from the Crandall '616 patent does not expressly disclose nor does it suggest:

1. that the initial point (x_1, y_1), the field F_{pk}, and/or curve parameter "a" are stored into the separate source 813 by:
   a. the sender;
   b. the receiver; or
   c. an unidentified trusted third party; or
2. how the initial point (x_1, y_1), the field F_{pk}, and/or curve parameter "a" become publicly known.

Legal Principles Applicable to Rejections Under 35 U.S.C. 102(b)

> [F]or anticipation under 35 U.S.C. § 102, the reference must teach **every aspect** of the claimed invention either explicitly or impliedly. Any feature not directly taught must be inherently present. Manual of Patent Examining Procedure ("MPEP") Eighth Edition Revision 2, May 2004, § 706.02, p. 700-21 (Emphasis supplied)

"Anticipation under 35 U.S.C. § 102 requires the disclosure in a single piece of prior art of each and every limitation of a claimed invention." Rockwell International Corporation v. The United States, 147 F.3d 1358, 1363, 47 USPQ2d 1027, 1031 (Fed. Cir. 1998) citing National Presto Indus. v. West Bend Co., 76 F.3d 1184, 1189, 37 USPQ2d 1685, 1687 (Fed. Cir. 1966).

Argument

Applicant respectfully submits that for the reasons specifically set forth in detail below independent claims 40 and 27 traverse all bases for rejection appearing in the Office Action dated August 25, 2004. Furthermore, because independent claim 27 traverses all bases for rejection, independent claims 1 and 14 similarly traverse rejection. Finally, because claims 2-13, 15-26, 28-39 and 41 respectively depend from independent claims 1, 14, 27 and 40, and because those independent claims traverse rejection, dependent claims 2-13, 15-26, 28-39 and 41 also traverse any and all rejections appearing in the August 25, 2004, Office Action.

Claim 40 Traverses Rejection

Digital signature independent claim 40 has been finally rejected under 35 U.S.C. § 102(b) as being anticipated by the Crandall '616 patent.

Regarding publicly known parameters other than ourPub and theirPub used in the Crandall '616 patent's elliptic curve encryption/decryption and/or digital signature authentication cryptosystems, i.e.

1. the:

    a. "bit-depth" q; or

b. a fast class number $p = 2^q - C$ where q is the bit depth of the encryption scheme;

2. the field determining parameter k, so that F_{pk} will be the field;

3. the initial point on the elliptic curve:

a. (x_1, y_1); or

b. (X_1/Z);

4. the field F_{pk}; or

5. the curve-defining parameter "a";

Applicant respectfully submits that nothing in that reference's drawings and/or text supports the March 29, 2005, Office Action's unjustified summary declaration that the Crandall '616 patent implicitly discloses that the sender stores the plurality of public elliptic curve cryptosystem parameters. Rather, **Exhibits A-C hereto attached hereto and the analyses of the Crandall '616 patent set forth above prove that the reference's text contradicts the March 29, 2005, Office Action's summary declaration that either the sender or the receiver stores the parameters into the separate source 813, i.e. store them into a publicly accessibly repository**.

In addition to claim 40's allowability because the Crandall '616 patent fails to disclose, either expressly or implicitly, the sender's storage of more than one (1) quantity, i.e. more than ourPub, into the public source 813, claim 40 is also allowable

because the Crandall patent discloses evaluating only two (2) equations and one (1) comparison as contrasted with independent claim 40's express requirement, as demonstrated below, for evaluating four (4) equations and performing two (2) independent comparisons.

The Crandall '616 patent, beginning in column 16 at line 48 and continuing through column 17, line 28, discloses computing in two different ways elliptic points Q and R. The Crandall '616 patent then discloses that a digital signature attached to an encrypted message is authenticated if Q equals R.

The present application beginning on page 22 at line 6 and continuing through page 24, line 13, discloses evaluating for each two (2) independent verification relationships two (2) expressions each of which is respectively located on opposite sides of a comparison symbol "$\stackrel{?}{=}$". A reformulation of the two (2) independent verification relationships, which appear in the present application on page 23 in lines 13 through 16, into the symbology of the Crandall '616 patent appears below.

1. $$\mathrm{m}^{((\mathbf{a}.\,p)\wedge \mathrm{n})\,(e \times (e \times a)) + a \times p)\,.\,((e \times a) + (e\,.\,a)\wedge \mathrm{n}\; e))} = \mathbf{Q}_1$$

$$\stackrel{?}{=} \mathbf{R}_1 = \mathrm{m}^{-a \times (e \times a)\,.\,p}\,\mathrm{m}^{-a \times e\,(e\,.\,a)\wedge \mathrm{n}\,.\,p}$$

2. $$\mathrm{m}^{((\mathbf{a}.\,p)\wedge \mathrm{n})\,(e \times (e \times a)) + a \times p)\,.\,((e\,.\,\alpha \times a)\wedge \mathrm{n} + (e\,.\,\alpha \times a)\,(e \times a) \times (\alpha \times a) \times e} = \mathbf{Q}_2$$

$$\stackrel{?}{=} \mathbf{R_2} = m^{-((\boldsymbol{e} \cdot \alpha \times \boldsymbol{a})^\wedge n + (\boldsymbol{e} \cdot \alpha \times \boldsymbol{a})((\boldsymbol{e} \times \boldsymbol{a}) \times ((\alpha \times \boldsymbol{a}) \times \boldsymbol{e}) \times \boldsymbol{a} \cdot \boldsymbol{p}}$$

The preceding reformulation of the two (2) independent verification relationships into the symbology of the Crandall '616 patent clearly requires:

1. computing Q_1, R_1, Q_2 and R_2; and
2. comparing:
   a. $Q_1 \stackrel{?}{=} R_1$; and
   b. $Q_2 \stackrel{?}{=} R_2$.

Thus, the allegation appearing on page 4 in lines 9 through 11 of the March 29, 2005, Office Action, that the Crandall '616 patent "expressly teaches [2.] at least two expressions being evaluated by the receiver using a plurality of public quantities" fails to disclose or to even suggest comparing the results obtained by evaluating expressions of at least two (2) different verification relationships as required by independent claim 40.

For the preceding reasons, the Crandall '616 patent does not, in the terminology of the present application, disclose nor does it suggest comparing the results obtained by evaluating expressions of at least two (2) different verification relationships. **Because the Crandall '616 patent fails to disclose or even suggest comparing the results obtained by evaluating expressions of at least two (2) different verification relationships which the text of independent**

claim 40 expressly requires, Applicant respectfully submits that for this second reason claim 40 traverses rejection under 35 U.S.C. § 102(b) based upon the Crandall '616 patent.

Thus, as best summarized in the table attached to the January 28, 2005, response to the August 24, 2005, Office Action as Exhibit B which is hereby incorporated by reference as though fully set forth here, and as explained in greater detail above, regarding independent claim 40 the Crandall '616 fails to disclose or to suggest:

1. that the "sender 1201" stores **a plurality of public quantities** into the "public source 813" which the "receiver 1202" retrieves during digital signature authentication;
2. at least **two (2) expressions are evaluated** by the receiver **using a plurality of public quantities**; and
3. comparing **the at least two (2) expressions evaluated using a plurality of public quantities**.

Rather, for digital signature authentication for the reasons set forth above the Crandall '616 patent expressly discloses that:

1. the **"sender 1201" stores a single x-coordinate**, ourPub, into the "public source 813" rather than a plurality of quantities;

2. the "receiver 1202" **evaluates** with the "hasher 1206" an expression to obtain **the quantity R using the single x-coordinate**, ourPub, received from the "public source 813;"

3. the "receiver 1202" **evaluates** with the "elliptic multiplier 806" an expression to obtain **the quantity Q without using any quantity received from the "public source 813**;" and

4. **authenticates** the digital signature **by comparing**:
   a. **the quantity R evaluated using the single x-coordinate public quantity**, ourPub, received from the "public source 813;" and
   b. **the quantity Q evaluated without using any quantity received from the "public source 813."**

For the preceding reasons, there exists at least two (2) essential differences between the disclosure of the Crandall '616 patent and the invention encompassed by pending independent claim 40. Therefore, because the Crandall '616 patent fails to disclose each and every limitation expressly required by the text of pending independent claim 40, Applicant respectfully:

1. submits that independent claim 40 traverses rejection under 35 U.S.C. § 102(b) based upon the Crandall '616 patent; and

2. requests that the rejection of independent claim 40 appearing in the August 25, 2004, Office Action be withdrawn.

Claim 27 Traverses Rejection

Applicant acknowledges that the Hellman, et al. patent discloses everything recited in the preamble of independent claim 27, and everything recited in element "b. a cryptographic device" in the body of that claim. Applicant further acknowledges that the Hellman, et al. patent implicitly discloses in column 8 at lines 21 and 22 storing a plurality of public quantities, i.e. the signals q and a, in a publicly accessible repository. However, Applicant respectfully submits that the Hellman, et al. patent fails to disclose or to suggest either sub-element "i" or sub-element "ii" of element "a. ports" in the body of independent claim 27.

Considering sub-element "i" of element "a. ports" in the body of independent claim 27, sending converser 12 depicted in FIG. 1 of the Hellman, et al. patent:

1. produces the signals q and a;[9] and
2. implicitly makes q and a publicly known.[10]

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[9] See the Hellman, et al. patent in column 4 at lines 1 through 11.

[10] See the Hellman, et al. patent in column 8 at lines 21 through 22.

Then, the receiving converser 12 in accordance with the disclosure of the Hellman, et al. patent in column 4 at lines at lines 1 through 51:

1. obtains Y_2 by transforming in the secure key generator 22 the receiver's private key X_2 using the public signals q and a; and

2. transmits Y_2 to the sending converser 11.

Considering first sub-element "i" of element "a. ports" in the body of independent claim 27, sub-element "i" initially requires a port that:

i. when the **cryptographic unit** **is to receive** the cyphertext message M[, i.e. converser 12], for:

   (1) **storing plurality of public quantities in a publicly accessible repository**.

Since as set forth above **only the Hellman, et al. patent's sending converser 11, after generating q and a, may store them into a publicly accessible repository, the Hellman, et al. patent fails to disclose or to suggest** portion (1) of sub-element "i" of element "a. ports" excerpted above from independent claim 27. Stated in a different way, **the Hellman, et al. patent fails to disclose the receiving converser 12 storing anything into a publicly accessible repository** as required by portion (1) of sub-element "i" of element "a. ports."

Considering again sub-element "i" of element "a. ports" in the body of independent claim 27, sub-element "i" subsequently requires a port that:

i. when the **cryptographic unit** **is to receive**[, i.e. converser 12] the cyphertext message M, for:

* * *

(2) receiving via the communication channel I a plurality of sender's quantities from a sending cryptographic unit[, i.e. q and a], and **the receiving cryptographic unit** using the plurality of sender's quantities[, i.e. q and a,] **and at least some of the plurality of public quantities** in computing:

(a) at least one receiver's quantity which said receiving cryptographic unit transmits via the communication channel I to said sending cryptographic unit[, i.e. Y_2]; and

(b) the key K.

One analysis of sub-element "i, (2)" of element "a. ports" guesses that q and a disclosed in the Hellman, et al. patent constitutes the "sender's quantities" appearing in the text of independent claim 27's sub-element "i, (2)" of element "a. ports."

However, **if q and a disclosed in the Hellman, et al. patent constitutes the "plurality of sender's quantities" appearing in the text of independent claim 27's sub-element "i, (2)" of element "a. ports,"** **then q and a can not also be "at least some of the plurality of public quantities"** appearing in the text of independent claim 27's sub-element "i, (2)" of element "a. ports." Conversely, **if one hypothesizes that q and a are the "at least some of the plurality of public quantities"** appearing in the text of independent claim 27's sub-element "i, (2)" of element "a. ports," **then q and a can not also be the "plurality of sender's quantities [received] from a sending cryptographic unit"** appearing in the text of independent claim 27's sub-element "i, (2)" of element "a. ports." **Stated a little more concisely, elements q and a disclosed in the Hellman et al. patent must be either, but not both, of:**

1. **the "plurality of sender's quantities [received] from a sending cryptographic unit;" or**
2. **the at least some of the plurality of public quantities**;

required by sub-element "i, (2)" of element "a. ports" in independent claim 27.

Considering now sub-element "ii" of element "a. ports" in the body of independent claim 27, sub-element "ii" requires a port that:

ii. when the cryptographic unit is to send the cyphertext message M[, i.e. converser 11], **for retrieving the plurality of public quantities from the publicly accessible repository**, the sending cryptographic unit **using the retrieved plurality of public quantities in computing**:

(1) the plurality of sender's quantities which the sending cryptographic unit transmits via the communication channel I to the receiving cryptographic unit; and

(2) after receiving via the communication channel I the receiver's quantity[, Y_2,] from the receiving cryptographic unit, the key K.

Since as set forth above **the Hellman, et al. patent's sending converser 11, receives only Y_2 from the receiving converser 12, the Hellman, et al. patent fails to disclose or to suggest** sub-element "ii" of element "a. ports" excerpted above independent claim 27. Stated in a different way, **the Hellman, et al. patent's sending converser 12, does retrieve anything from a publicly accessible repository**, no less a plurality of public quantities required by sub-element "ii" of element "a. ports".

The August 25, 2004, Office Action rejects independent claim 27 under 35 U.S.C. § 103(a) based upon the Hellman, et al. patent

in view of Schneier. Paragraph no. 12 of the August 25, 2004, Office Action explains the pertinence of Schneier as follows.

> Hellman does not expressly disclose storing a plurality of public quantities in a publicly accessible repository. However, the variables q and a used in Diffie-Hellman key exchange are public variables within a public-key cryptosystem, which enables these public variables to be published in a public repository as taught by Schneier. See Schneier, page 32, 2nd paragraph; page 515, 'Key Exchange Without Exchanging Keys'. Furthermore, a third party repository acts as a disinterested member of a communications system and can ensure the certification, renewal and cancellation of public information. See Schneier, page 23, 'Arbitrated Protocols'. It would be obvious to one of ordinary skill in the art at the time the invention was made to store the plurality of public quantities in a public accessible repository and retrieve the plurality of public quantities from the public accessible repository for secure key exchange to simplify the key exchange process. See Schneier, page 32, 3rd paragraph. The aforementioned covers claim 27.

Applicant notes that while the Hellman, et al. patent may not "expressly disclose storing a plurality of public quantities in a publicly accessible repository" as stated in the preceding excerpt from the August 25, 2004, Office Action, **Applicant respectfully submits that lines 21 and 22 in column 8 of the Hellman, et al. patent implicitly and necessarily make such a disclosure**. The text in lines 21 and 22 in column 8 of the Hellman, et al. patent states that:

> signals q and a may be public knowledge rather than generated by the key source 25.

While the preceding excerpt from the Hellman, et al. patent doesn't expressly mention a publicly accessible repository, for q and a to

be publicly known the only alternative to them being available in a publicly accessible repository is for them to be part of each individual's instinct at birth. Since the latter alternative appears highly unlikely if not impossible, it appears that making q and a "public knowledge" necessarily requires that they be made available in some sort of publicly accessible repository.

Because, for the preceding reasons the Hellman, et al. patent implicitly discloses "storing a plurality of public quantities in a publicly accessible repository," Applicant respectfully submits that **Schneier** as applied to claim pending in the present application by paragraph 12 **adds nothing to the disclosure of the Hellman, et al. patent**. Since Schneier as applied to claim pending in the present application by paragraph 12 adds nothing to the disclosure of the Hellman, et al. patent, and since for the reasons set forth above the Hellman, et al. patent fails to disclose or to suggest either sub-element i. or ii. of element b. in independent claim 27, Applicant respectfully:

1. submits that independent claim 27 traverses rejection under 35 U.S.C. § 103(a) based upon the Hellman, et al. patent in view of Schneier; and
2. requests that the rejection of independent claim 27 appearing in the August 25, 2004, Office Action be withdrawn.

Independent Claims 1 & 14 Traverse Rejection

Applicant agrees respectively with paragraphs 18 and 19 in the August 25, 2004, Office Action that:

1. independent claim 1 is a method claim corresponding to independent claim 27; and
2. independent claim 14 is a system claim corresponding to independent claim 27.

Therefore, because for the reasons set forth above independent claim 27 traverses rejection under 35 U.S.C. § 103(a) based upon the Hellman, et al. patent in view of Schneier, Applicant respectfully submits that independent claims 1 and 14 also traverse rejection based upon that combination of references.

Dependent Claims 2-13, 15-26, 28-39 and 41 All Traverse Rejection

Dependent claims 2-13, 15-26, 28-39 and 41 respectively depend from independent claims 1, 14, 27 and 40. Because for the reasons set forth above independent claims 1, 14, 27 and 40 traverse rejection on the bases set forth in the August 25, 2004, Office Action, Applicant respectfully submits that claims 2-13, 15-26, 28-39 and 41, which depend respectively from independent claims 1, 14, 27 and 40, also traverse rejection for any and all reasons appearing in the August 25, 2004, Office Action.

Conclusion

Because for the reasons set forth above independent claim 40 traverses rejection under 35 U.S.C. § 102(b) based upon the Crandall '616 patent, Applicant respectfully requests:

1. that the rejection of independent claim 40 together with the rejection of claim 41 depending therefrom based upon the Crandall '616 patent be withdrawn; and
2. that claims 40 and 41 pass promptly to issue.

Because for the reasons set forth above independent claim 27 traverses rejection under 35 U.S.C. § 103(a) based upon the Hellman, et al. patent in view of Schneier, Applicant respectfully requests:

1. that the rejection of independent claim 27 together with any and all rejections of claim 28-39 depending therefrom based upon that combination of references, either alone or in further combination with any other reference(s), be withdrawn; and
2. that claims 27 through 30 pass promptly to issue.

Because for the reasons set forth above independent claim 1 traverses rejection under 35 U.S.C. § 103(a) based upon the Hellman, et al. patent in view of Schneier, Applicant respectfully requests:

1. that the rejection of independent claim 1 together with any and all rejections of claim 2-13 depending therefrom based upon that combination of references, either alone or further in combination with any other reference(s), be withdrawn; and
2. that claims 1 through 13 pass promptly to issue.

Because for the reasons set forth above independent claim 14 traverses rejection under 35 U.S.C. § 103(a) based upon the Hellman, et al. patent in view of Schneier, Applicant respectfully requests:

1. that the rejection of independent claim 14 together with any and all rejections of claim 15-26 depending therefrom based upon that combination of references, either alone or further in combination with any other reference(s), be withdrawn; and
2. that claims 14 through 26 pass promptly to issue.

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For the preceding reasons, the Applicant respectfully requests favorable reconsideration and allowance of claims 1-41 presently pending in this application.

Respectfully submitted,

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INSTANCE LIST
United States Patent no. 5,581,616
(The Crandall '616 Patent)

	Col.-Line	Col.-Line	Col.-Line
public	1 - 41	1 - 42	1 - 51
publicly	1 - 52	1 - 58	2 - 32
	2 - 59	2 - 63	3 - 2
	3 - 6	3 - 7	3 - 8
	3 - 10	3 - 11	3 - 15
	3 - 16	3 - 18	3 - 20
	3 - 23	3 - 28	3 - 29
	3 - 30	3 - 32	3 - 37
	3 - 39	3 - 40	3 - 56
	4 - 12	4 - 13	4 - 20
	4 - 33	4 - 38	4 - 50
	5 - 2	5 - 5	5 - 17
	5 - 25	5 - 28	5 - 47
	5 - 53	5 - 64	6 - 22
	6 - 24	7 - 8	7 - 26
	7 - 27	7 - 29	7 - 32
	7 - 57	8 - 2	8 - 6
	8 - 10	8 - 15	8 - 45
	12 - 63	12 - 64	13 - 2
	13 - 9	13 - 21	14 - 16
	14 - 21	14 - 25	14 - 29
	15 - 14	15 - 16	15 - 19
	15 - 20	15 - 22	15 - 24
	16 - 7	16 - 8	16 - 24
	17 - 50	17 - 27	19 - 54
	19 - 55	19 - 60	20 - 19
	20 - 20	20 - 32	20 - 55
separate source 813	12 - 63	19 - 54	
public source 813	13 - 2	13 - 22	19 - 60
	20 - 19		
source 813	19 - 42	20 - 32	
ourPub sender's	3 - 11	5 - 53	8 - 6
public key	8 - 8	8 - 15	8 - 25
	12 - 64	13 - 21	13 - 23
	14 - 22	14 - 24	14 - 29
	14 - 35	15 - 23	15 - 29
	16 - 24	16 - 59	16 - 65
	17 - 1	17 - 7	17 - 9
	17 - 35	17 - 43	17 - 51
	18 - 24	18 - 56	18 - 58
	19 - 42	19 - 55	20 - 19
	20 - 21	20 - 32	20 - 52

EXHIBIT A

		Col.-Line	Col.-Line	Col.-Line
myPub	sender's public key	3 - 19	3 - 21	3 - 59
		3 - 61	5 - 64	
theirPub	receiver's public key	3 - 12	3 - 26	3 - 39
		3 - 42	3 - 44	8 - 10
		3 - 12	8 - 15	8 - 20
		12 - 64	13 - 9	13 - 10
		14 - 26	14 - 28	14 - 29
		14 - 32	15 - 24	15 - 27
		19 - 55		

Hi-Lited Text of
United States Patent no. 5,581,616
(The Crandall '616 Patent)

EXHIBIT B

METHOD AND APPARATUS FOR DIGITAL SIGNATURE AUTHENTICATION

This is a continuation in part of U.S. patent application Ser. No. 08/167,408 filed Dec. 14, 1993 (now issued as U.S. Pat. No. 5,463,690) which is a continuation of U.S. patent application Ser. No. 07/955,479 filed Oct. 2, 1992 (now issued as U.S. Pat. No. 5,271,061) which is a continuation of U.S. application Ser. No. 07/761,276 filed Sep. 17, 1991 (now issued as U.S. Pat. No. 5,159,632).

BACKGROUND OF THE PRESENT INVENTION

1. Field of the Invention

This invention relates to the field of cryptographic systems.

2. Background Art

A cryptographic system is a system for sending a message from a sender to a receiver over a medium so that the message is "secure", that is, so that only the intended receiver can recover the message. A cryptographic system converts a message, referred to as "plaintext" into an encrypted format, known as "ciphertext." The encryption is accomplished by manipulating or transforming the message using a "cipher key" or keys. The receiver "decrypts" the message, that is, converts it from ciphertext to plaintext, by reversing the manipulation or transformation process using the cipher key or keys. So long as only the sender and receiver have knowledge of the cipher key, such an encrypted transmission is secure.

A "classical" cryptosystem is a cryptosystem in which the enciphering information can be used to determine the deciphering information. To provide security, a classical cryptosystem requires that the enciphering key be kept secret and provided to users of the system over secure channels. Secure channels, such as secret couriers, secure telephone transmission lines, or the like, are often impractical and expensive.

A system that eliminates the difficulties of exchanging a secure enciphering key is known as "public key encryption." By definition, a public key cryptosystem has the property that someone who knows only how to encipher a message cannot use the enciphering key to find the deciphering key without a prohibitively lengthy computation. An enciphering function is chosen so that once an enciphering key is known, the enciphering function is relatively easy to compute. However, the inverse of the encrypting transformation function is difficult, or computationally infeasible, to compute. Such a function is referred to as a "one way function" or as a "trap door function." In a public key cryptosystem, certain information relating to the keys is public. This information can be, and often is, published or transmitted in a non-secure manner. Also, certain information relating to the keys is private. This information may be distributed over a secure channel to protect its privacy, (or may be created by a local user to ensure privacy).

A block diagram of a typical public key cryptographic system is illustrated in FIG. **1**. A sender represented by the blocks within dashed line **100** sends a plaintext message Ptxt to a receiver, represented by the blocks within dashed line **115**. The plaintext message is encrypted into a ciphertext message C, transmitted over some transmission medium and decoded by the receiver **115** to recreate the plaintext message Ptxt.

The sender **100** includes a cryptographic device **101**, a secure key generator **102** and a key source **103**. The key source **103** is connected to the secure key generator **102** through line **104**. The secure key generator **102** is coupled to the cryptographic device **101** through line **105**. The cryptographic device provides a ciphertext output C on line **106**. The secure key generator **102** provides a key output on line **107**. This output is provided, along with the ciphertext message **106**, to transmitter receiver **109**. The transmitter receiver **109** may be, for example, a computer transmitting device such as a modem or it may be a device for transmitting radio frequency transmission signals. The transmitter receiver **109** outputs the secure key and the ciphertext message on an insecure channel **110** to the receiver's transmitter receiver **111**.

The receiver **115** also includes a cryptographic device **116**, a secure key generator **117** and a key source **118**. The key source **118** is coupled to the secure key generator **117** on line **119**. The secure key generator **117** is coupled to the cryptographic device **116** on line **120**. The cryptographic device **116** is coupled to the transmitter receiver **111** through line **121**. The secure key generator **117** is coupled to the transmitter receiver **111** on lines **122** and **123**.

In operation, the sender **100** has a plaintext message Ptxt to send to the receiver **115**. Both the sender **100** and the receiver **115** have cryptographic devices **101** and **116**, respectively, that use the same encryption scheme. There are a number of suitable cryptosystems that can be implemented in the cryptographic devices. For example, they may implement the Data Encryption Standard (DES) or some other suitable encryption scheme.

Sender and receiver also have secure key generators **102** and **117**, respectively. These secure key generators implement any one of several well known public key exchange schemes. These schemes, which will be described in detail below, include the Diffie-Hellman scheme, the RSA scheme, the Massey-Omura scheme, and the ElGamal scheme.

The sender **100** uses key source **103**, which may be a random number generator, to generate a private key. The private key is provided to the secure key generator **102** and is used to generate an encryption key e_K. The encryption key e_K is transmitted on lines **105** to the cryptographic device and is used to encrypt the plaintext message Ptxt to generate a ciphertext message C provided on line **106** to the transmitter receiver **109**. The secure key generator **102** also transmits the information used to convert to the secure key from key source **103** to the encryption key e_K. This information can be transmitted over an insecure channel, because it is impractical to recreate the encryption key from this information without knowing the private key.

The receiver **115** uses key source **118** to generate a private and secure key **119**. This private key **119** is used in the secure key generator **117** along with the key generating information provided by the sender **100** to generate a deciphering key D_K. This deciphering key D_K is provided on line **120** to the cryptographic device **116** where it is used to decrypt the ciphertext message and reproduce the original plaintext message.

The Diffie-Hellman Scheme

A scheme for public key exchange is presented in Diffie and Hellman, "New Directions in Cryptography," IEEE Trans. Inform. Theory, vol. IT-22, pp. 644–654, November 1976 (The "DH" scheme). The DH scheme describes a public key system based on the discrete exponential and logarithmic functions. If "q" is a prime number and "a" is a primitive element, then X and Y are in a 1:1 correspondence for $1 \leq X, Y \leq (q-1)$ where $Y=a^X \bmod q$, and $X=\log_a Y$ over the finite field. The first discrete exponential function is

easily evaluated for a given a and X, and is used to compute the public key Y. The security of the Diffie-Hellman system relies on the fact that no general, fast algorithms are known for solving the discrete logarithm function $X=\log_a Y$ given X and Y.

In a Diffie-Hellman system, a directory of public keys is published or otherwise made available to the public. A given public key is dependent on its associated private key, known only to a user. However, it is not feasible to determine the private key from the public key. For example, a sender has a public key, referred to as "ourPub". A receiver has a public key, referred to here as "theirPub". The sender also has a private key, referred to here as "myPri". Similarly, the receiver has a private key, referred to here as "theirPri".

There are a number of elements that are publicly known in a public key system. In the case of the Diffie-Hellman system, these elements include a prime number p and a primitive element g. p and g are both publicly known. Public keys are then generated by raising g to the private key power (mod p). For example, a sender's public key myPub is generated by the following equation:

$$myPub=g^{myPri} \pmod p \quad \text{Equation (1)}$$

Similarly, the receiver's public key is generated by the equation:

$$theirPub=g^{theirPri} \pmod p \quad \text{Equation (2)}$$

Public keys are easily created using exponentiation and modulo arithmetic. As noted previously, public keys are easily obtainable by the public. They are published and distributed. They may also be transmitted over non-secure channels. Even though the public keys are known, it is very difficult to calculate the private keys by the inverse function because of the difficulty in solving the discrete log problem.

FIG. **2** illustrates a flow chart that is an example of a key exchange using a Diffie-Hellman type system. At step **201**, a prime number p is chosen. This prime number p is public. Next, at step **202**, a primitive root g is chosen. This number g is also publicly known. At step **203** an enciphering key e_K is generated, the receiver's public key (theirPub) is raised to the power of the sender's private key (myPri). That is:

$$(theirPub)^{myPri} \pmod p \quad \text{Equation (3)}$$

We have already defined theirPub equal to $g^{theirPri}$ (mod p). Therefore Equation 3 can be given by:

$$(g^{theirPri})^{myPri} \pmod p \quad \text{Equation (4)}$$

This value is the enciphering key e_K that is used to encipher the plaintext message and create a ciphertext message. The particular method for enciphering or encrypting the message may be any one of several well known methods. Whichever encrypting message is used, the cipher key is the value calculated in Equation 4. The ciphertext message is then sent to the receiver at step **204**.

At step **205**, the receiver generates a deciphering key D_K by raising the public key of the sender (myPri) to the private key of the receiver (theirPri) as follows:

$$D_K=(myPub)^{theirPri} \pmod p \quad \text{Equation (5)}$$

From Equation 1, myPub is equal to g^{myPri} (mod p). Therefore:

$$D_K=(g^{myPri})^{theirPri} \pmod p \quad \text{Equation (6)}$$

Since $(g^A)^B$ is equal to $(g^B)^A$, the encipher key e_K and the deciphering key D_K are the same key. These keys are referred to as a "one-time pad." A one-time pad is a key used in enciphering and deciphering a message.

The receiver simply executes the inverse of the transformation algorithm or encryption scheme using the deciphering key to recover the plaintext message at step **206**. Because both the sender and receiver must use their private keys for generating the enciphering key, no other users are able to read or decipher the ciphertext message. Note that step **205** can be performed prior to or contemporaneously with any of steps **201–204**.

RSA

Another public key cryptosystem is proposed in Rivest, Shamir and Adelman, "On Digital Signatures and Public Key Cryptosystems," Commun. Ass. Comput. Mach., vol. 21, pp. 120–126, February 1978 (The "RSA" scheme). The RSA scheme is based on the fact that it is easy to generate two very large prime numbers and multiply them together, but it is much more difficult to factor the result, that is, to determine the very large prime numbers from their product. The product can therefore be made public as part of the enciphering key without compromising the prime numbers that effectively constitute the deciphering key.

In the RSA scheme a key generation algorithm is used to select two large prime numbers p and q and multiply them to obtain n=pq. The numbers p and q can be hundreds of decimal digits in length. Then Euler's function is computed as $\phi(n)=(p-1)(q-1)$. ($\phi(n)$ is the number of integers between 1 and n that have no common factor with n). $\phi(n)$ has the property that for any integer a between 0 and n−1 and any integer k, $a^{k\phi(n)+1}=a \pmod n$.

A random number E is then chosen between 1 and $\phi(n)-1$ and which has no common factors with $\phi(n)$. The random number E is the enciphering key and is public. This then allows $D=E^{-1} \bmod \phi(n)$ to be calculated easily using an extended version of Euclid's algorithm for computing the greatest common divisor of two numbers. D is the deciphering key and is kept secret.

The information (E, n) is made public as the enciphering key and is used to transform unenciphered, plaintext messages into ciphertext messages as follows: a message is first represented as a sequence of integers each between 0 and n−1. Let P denote such an integer. Then the corresponding ciphertext integer is given by the relation $C=P^E \bmod n$. The information (D, n) is used as the deciphering key to recover the plaintext from the ciphertext via $P=C^D \bmod n$. These are inverse transformations because $C^D=P^{ED}=P^{k\phi(n)+1}=P$.

MASSEY-OMURA

The Massey-Omura cryptosystem is described in U.S. Pat. No. 4,567,600. In the Massey cryptosystem, a finite field F_q is selected. The field F_q is fixed and is a publicly known field. A sender and a receiver each select a random integer e between 0 and q−1 so that the greatest common denominator G. C. D. (e, q−1)=1. The user then computes its inverse $D=e^{-1}$ mod q−1 using the euclidian algorithm. Therefore, De=1 mod q−1.

The Massey-Omura cryptosystem requires that three messages be sent to achieve a secure transmission. Sender A sends message P to receiver B. Sender A calculates random number e_A and receiver B calculates random number e_B. The sender first sends the receiver the element P^{e_A}. The receiver is unable to recover P since the receiver does not know e_A. Instead, the receiver raises the element to his own private key e_B and sends a second message $P^{e_A e_B}$ back to the sender. The sender then removes the effect of e_A by raising the element to the $D_{A\text{-}th}$ power and returns P_{eB} to the receiver B. The receiver B can read this message by raising the element to the $D_{B\text{-}th}$ power.

ELGAMAL CRYPTOSYSTEM

The ElGamal public key cryptosystem utilizes a publicly known finite field F_q and an element g of F^*_q. Each user randomly chooses an integer a=to a_A in the range 0>a>q−1. The integer a is the private deciphering key. The public enciphering key is the element g^a F_q. To send a message represented by P to a user A, an integer K is randomly chosen. A pair of elements of F_q, namely (g^K, pg^{aK}) are sent to A. The plaintext message Ptxt is encrypted with the key g^{aK}. The value g^K is a "clue" to the receiver for determining the plaintext message Ptxt. However, this clue can only be used by someone who knows the secure deciphering key "a". The receiver A, who knows "a", recovers the message P from this pair by raising the first element $gK^{a/h}$ and dividing the result into the second element.

ELLIPTIC CURVES

Another form of public key cryptosystem is referred to as an "elliptic curve" cryptosystem. An elliptic curve cryptosystem is based on points on an elliptic curve E defined over a finite field F. Elliptic curve cryptosystems rely for security on the difficulty in solving the discrete logarithm problem. An advantage of an elliptic curve cryptosystem is there is more flexibility in choosing an elliptic curve than in choosing a finite field. Nevertheless, elliptic curve cryptosystems have not been widely used in computer-based public key exchange systems due to their computational intensiveness. Computer-based elliptic curve cryptosystems are slow compared to other computer public key exchange systems. Elliptic curve cryptosystems are described in "A Course in Number Theory and Cryptography" (Koblitz, 1987, Springer-Verlag, New York).

AUTHENTICATION

In addition to protecting the contents of a transmitted message, it is also desired to provide a way to determine the "authenticity" of the message. That is, is the message actually from the purported sender. A scheme for accomplishing this is to append a so-called "digital signature" to the message. One such scheme is described in Koblitz, supra. The enciphering transformation f_A is used to send a message to user A and f_B is the enciphering transformation used to send a message to user B. User A provides a "signature" P that may include some specific information, such as the time the message was sent or an identification number. User A transmits the signature as $f_B f_A^{-1}$ (P). When user B deciphers the message using f_B^{-1}, the entire message is decoded into plaintext except the signature portion, which remains f_A^{-1} (P). User B then applies user A's public key f_A to obtain P. Since P could only have been encrypted by user A (because only user A knows f_A^{-1}) user B can assume that the message was sent by user A.

Another scheme of digital signature authentication is a generalization of the ElGamal discrete logarithm scheme, using elliptic algebra. Assume a public key ourPub generated with a function of a private key ourPri. The signature is generated by first choosing a random integer m of approximately q bits. Next a point $P=m°(X_1/1)$ is computed. A message digest function M is used to compute an integer u that is a function of m, ourPri, and the digested version of the ciphertext message and the computed point P. The computed pair (u, P) is transmitted as the signature.

At the receiving end, the u value of the signature is used to compute the point $Q=u°(X_1/1)$. A point R is calculated using P, the digested version of the ciphertext message and P, and myPub. If R and Q do not compare exactly, the signature is not valid (not genuine). The security of this scheme relies on the computational infeasability of breaking the elliptic logarithm operation or the hash function M. A disadvantage of this scheme is that it is computationally intensive, making it complex and slow in operation.

SUMMARY OF THE INVENTION

The present invention improves speed and reduces complexity in a digital signature scheme that uses elliptic algebra. The signature scheme generates two points that are compared. If the points do not match, the signature is not authentic. The present invention reduces computations by comparing only the x coordinates of the two generated points. The invention provides a scheme for deducing the possible values of the x-coordinate of a sum of two points using only the x coordinates of the original two points in question. The present invention provides a scheme that limits the possible solutions that satisfy the equation to two (the authentic signature and one other). Because of the large number of possible inauthentic solutions, the chance of a false authentic signature is statistically insignificant.

BRIEF DESCRIPTION OF THE DRAWINGS

FIG. 1 is a block diagram of a prior art public key exchange system.

FIG. 2 is a flow diagram of a prior art public key exchange transaction.

FIG. 3 is a flow diagram illustrating the key exchange of the present invention.

FIG. 4 is a block diagram of a computer system on which the present invention may be implemented.

FIG. 5 is a diagram illustrating the shift and add operations for performing mod p arithmetic using Mersenne primes.

FIG. 6 is a diagram illustrating the operations for performing mod p arithmetic using Fermat numbers.

FIG. 7 is a diagram illustrating the operations for performing mod p arithmetic using fast class numbers.

FIG. 8 is a block diagram of the present invention.

FIG. 9 is a flow diagram illustrating the operation of one embodiment of the present invention.

FIG. 10 is a flow diagram illustrating the generation of a digital signature using the present invention.

FIG. 11 is a flow diagram illustrating the authentication of a digital signature in the present invention.

FIG. 12 illustrates a block diagram for implementing the digital signature scheme of the present invention.

DETAILED DESCRIPTION OF THE INVENTION

An elliptic curve encryption scheme is described. In the following description, numerous specific details, such as number of bits, execution time, etc., are set forth in detail to provide a more thorough description of the present invention. It will be apparent, however, to one skilled in the art, that the present invention may be practiced without these specific details. In other instances, well known features have not been described in detail so as not to obscure the present invention.

A disadvantage of prior art computer-implemented elliptic curve encryption schemes is they are unsatisfactorily slow compared to other prior art computer-implemented encryption schemes. The modulo arithmetic and elliptical algebra operations required in a prior art elliptic curve cryptosystem require that divisions be performed. Divisions increase computer CPU (central processing unit) computational over-

head. CPU's can perform addition and multiplication operations more quickly, and in fewer processing steps, than division operations. Therefore, prior art elliptic curve cryptosystems have not been previously practical or desirable as compared to other prior art cryptosystems, such as Diffie-Hellman and RSA schemes.

The present invention provides methods and apparatus for implementing an elliptic curve cryptosystem for public key exchange that does not require explicit division operations. The advantages of the preferred embodiment of the present invention are achieved by implementing fast classes of numbers, inversionless parameterization, and FFT multiply mod operations.

Elliptic Curve Algebra

The elliptic curve used with the present invention is comprised of points $(x,y) \in F_{p^k} X F_{p^k}$ satisfying:

$$b\,y^2=x^3+a\,x^2+x \qquad \text{Equation (7)}$$

together with a "point at infinity" a.

Sender ("our") and recipient ("their") private keys are assumed to be integers, denoted:

ourPri, theirPri $\in$ Z Next, parameters are established for both sender and recipient. The parameters are: (mod p)

q, so that $p=2^q-C$ is a fast class number (q is the "bit-depth"). The value q is a publicly known value.

k, so that F_{p^k} will be the field, and where k is publicly known.

$(x_1, y_1) \in F_{p^k}$, the initial x-coordinate, which is publicly known.

$a \in F_{p^k}$, the curve-defining parameter (b is not needed). The value a is also publicly known.

The present invention uses an operation referred to as "elliptic multiplication" and represented by the symbol "°". The operation of elliptic multiplication can be described as follows:

An initial point (X_1, Y_1) on the curve of Equation 7 is defined. For the set of integers n, expression $n°(X_1, Y_1)$ denotes the point (X_n, Y_n) obtained via the following relations, known as adding and doubling rules.

$$X_{n+1}=((Y_n-Y_1)/(X_n-X_1))^2-X_1-X_n \qquad \text{Equation (8)}$$

$$Y_{n+1}=-Y_1+((Y_n-Y_1)/(X_n-X_1))(X_1-X_{n+1}) \qquad \text{Equation (9)}$$

When $(X_1, Y_1)=(X_n, Y_n)$, the doubling relations to be used are:

$$X_{n+1}=((3X_1^2+a)/2Y_1)^2-2X_1; \qquad \text{Equation (10)}$$

$$Y_{n+1}=-Y_1+((3X_1^2+a)/2Y_1)(X_1-X_{n+1}) \qquad \text{Equation (11)}$$

Because arithmetic is performed over the field F_{p^k}, all operations are to be performed (mod p). In particular, the division operation in equations 8 to 11 involve inversions (mod p).

Elliptic Curve Public Key Exchange

It is necessary that both sender and recipient use the same set of such parameters. Both sender and recipient generate a mutual one-time pad, as a particular x-coordinate on the elliptic curve.

In the following description, the terms "our" and "our end" refer to the sender. The terms "their" and "their end" refer to the receiver. This convention is used because the key exchange of the present invention may be accomplished between one or more senders and one or more receivers. Thus, "our" and "our end" and "their" and "their end" refers to one or more senders and receivers, respectively. The public key exchange of the elliptic curve cryptosystem of the present invention is illustrated in the flow diagram of FIG. 3.

Step 301

At our end, a public key is computed: ourPub $\in F_{p^k}$

$$\text{ourPub}=(\text{ourPri})°(x_1, y_1) \qquad \text{Equation (12)}$$

Step 302

At their end, a public key is computed: theirPub $\in F_{p^k}$

$$\text{theirPub}=(\text{theirPri})°(x_1, y_1) \qquad \text{Equation (13)}$$

Step 303

The two public keys ourPub and theirPub are published, and therefore known to all users.

Step 304

A one-time pad is computed at our end: ourPad $\in F_{p^k}$

$$\text{ourPad}=(\text{ourPri})°(\text{theirPub})=(\text{ourPri})°(\text{theirPri})°(x_1, y_1) \qquad \text{Equation (14)}$$

Step 305

A one-time pad is computed at their end: theirPad $\in F_{p^k}$

$$\text{theirPad}=(\text{theirPri})°(\text{ourPub})=(\text{theirPri})°(\text{ourPri})°(x_1, y_1) \qquad \text{Equation (15)}$$

The elements $(\text{theirPri})°(\text{ourPri})°(x_1, y_1)$ being part of a finite field, form an abelian group. Therefore, the order of operation of equations 14 and 15 can be changed without affecting the result of the equations. Therefore:

$$\text{ourPad}=(\text{ourPri})°(\text{theirPri})°(x_1, y_1)=(\text{theirPri})°(\text{ourPri})°(x_1, y_1)=\text{theirPad} \qquad \text{Equation (16)}$$

Since both the sender and receiver use the same one time pad, the message encrypted by the sender can be decrypted by the recipient, using the one time pad. (Note that step 305 can be executed prior to or contemporaneously with any of steps 301–304).

At step 306, the sender encrypts plaintext message Ptxt using ourPad, and transmits ciphertext message C to the receiver. At step 307, the receiver decrypts ciphertext message C to recover plaintext message Ptxt, using theirPad.

Fast Class Numbers

Elliptic curve cryptosystems make use of modulo arithmetic to determine certain parameters, such as public keys, one time pads, etc. The use of modulo arithmetic serves the dual purpose of limiting the number of bits in the results of equations to some fixed number, and providing security. The discrete log problem is asymmetrical in part because of the use of modulo arithmetic. A disadvantage of modulo arithmetic is the need to perform division operations. The solution to a modulo operation is the remainder when a number is divided by a fixed number. For example, 12 mod 5 is equal to 2. (5 divides into 12 twice with a remainder of 2, the remainder 2 is the solution). Therefore, modulo arithmetic requires division operations.

Special fast classes of numbers are used in the present invention to optimize the modulo arithmetic required in the enciphering and deciphering process by eliminating the need for division operations. The class of numbers used in the present invention is generally described by the form 2^q-C where C is an odd number and is relatively small, (e.g. no longer than the length of a computer word.

When a number is of this form, modulo arithmetic can be accomplished using shifts and adds only, eliminating the need for divisions. One subset of this fast class is known as "Mersenne" primes, and are of the form 2^q-1. Another class

that can be used with the present invention are known as "Fermat" numbers of the form 2^q+1, where q is equal to 2^m. Fermat numbers may be prime or not prime in the present invention.

The present invention utilizes elliptic curve algebra over a finite field F_{pk} where $p=2^q-C$ and p is a fast class number. Note that the equation 2^q-C does not result in a prime number for all values of q. and C For example, when q is equal to 4, and C is equal to 1, 2^q-C is equal to 15, not a prime. However, when q has a value of 2, 3, or 5, and C=1 the equation 2^q-C generates the prime numbers 3, 7, and 31.

The present invention implements elliptic curves over a finite field F_{pk} where p is 2^q-C is an element of a fast class of numbers. When practiced on a computer using binary representations of data, the use of fast class numbers allows the (mod p) operations to be accomplished using only shifts and adds. By contrast, the use of "slow" numbers requires that time consuming division operations be executed to perform (mod p) arithmetic. The following examples illustrate the advantage of fast class number (mod p) arithmetic.

EXAMPLE 1

base 10 (mod p) division

Consider the 32 bit digital number n, where n=11101011111000111001101O1 (In base 10 this number is 3,991,652,149).

Now consider n (mod p) where p is equal to 127. The expression n mod 127 can be calculated by division as follows:

```
          31430331
127 / 3991652149
      381
       181
       127
        546
        508
         385
         381
           42
            0
           421
           381
            404
            381
             239
             127
             112
```

The remainder 112 is the solution to n mod **127**.

EXAMPLE 2

Mersenne Prime (mod p) Arithmetic

In the present invention, when p is a Mersenne prime where $p=2^q-1$, the (mod p) arithmetic can be accomplished using only shifts and adds, with no division required. Consider again n (mod p) where n is 3,991,652,149 and p is 127. When p is 127, q is equal to 7, from $p=2^q-1$; $127=2^7-1=128-1=127$.

The (mod p) arithmetic can be accomplished by using the binary form of n, namely 11101101111010111100011100110101. Referring to FIG. **5**, the shifts and adds are accomplished by first latching the q least significant bits (LSB's) **501** of n, namely 0110101. The q LSB's **502** of the remaining digits, namely 0001110, are then added to q digits **501**, resulting in sum **503** (1000011). The next q LSB's **504** of n, (0101111), are added to sum **503**, generating sum **505**, (1110010). Bits **506** of n (1101111) are added to sum **505**, to result in sum **507**, (11100001).

The remaining bits **508** (1110), even though fewer in number than q bits, are added to sum **507** to generate sum **509** (11101111). This sum has greater than q bits. Therefore, the first q bits **510** (1101111) are summed with the next q bits **511** (in this case, the single bit 1), to generate sum **512** (1110000). This sum, having q or fewer bits, is the solution to n (mod p). $1110000=2^6+2^5+2^4=64+32+16=112$.

Thus, the solution 112 to n mod 127 is determined using only shifts and adds when an elliptic curve over a field of Mersenne primes is used. The use of Mersenne primes in conjunction with elliptic curve cryptosystems eliminates explicit divisions.

EXAMPLE 3

Fermat Number (mod p) Arithmetic

In the present invention, when p is a Fermat number where $p=2^q+1$, the (mod p) arithmetic can be accomplished using only shifts, adds, and subtracts (a negative add), with no division required. Consider again n (mod p) where n is 3,991,652,149 and where p is now 257. When p is 257, q is equal to 8, from $p=2^q+1$; $257=2^8+1=256+1=257$.

The (mod p) arithmetic can be accomplished by using the binary form of n, namely 11101101111010111100011100110101. Referring to FIG. **6**, the shifts and adds are accomplished by first latching the q (8) least significant bits (LSB's) **601** (00110101). The next q LSB's **602** of the remaining digits, namely 11000111, are to be subtracted from q digits **601**. To accomplish this, the 1's complement of bits **602** is generated and a 1 is added to the MSB side to indicate a negative number, resulting in bits **602'** (100111000). This negative number **602'** is added to bits **601** to generate result **603** (101101101). The next q LSB's **604** of n, (11101011), are added to sum **603**, generating result **605**, (1001011000). Bits **606** of n (11101101) are to be subtracted from result **605**. Therefore, the 1's complement of bits **606** is generated and a negative sign bit of one is added on the MSB side to generate bits **606'** (100010010). Bits **606'** is added to result **605**, to generate sum **607**, (1101101010).

Sum **607** has more than q bits so the q LSB's are latched as bits **608** (01101010). The next q bits (in this case, only two bits, **11**) are added to bits **608**, generating sum **610** (01101101). This sum, having q or fewer bits, is the solution to n (mod p). $01101101=2^6+2^5+2^3+2^2+2^0=64+32+8+4+1=109$.

EXAMPLE 4

Fast Class mod arithmetic

In the present invention, when p is a number of the class $p=2^q-C$, where C is and odd number and is relatively small, (e.g. no greater than the length of a digital word), the (mod p) arithmetic can be accomplished using only shifts and adds, with no division required. Consider again n (mod p) where n is 685 and where p is 13. When p is 13, q is equal to 4 and C is equal to 3, from $p=2^q-C$; $13=2^4-3=16-3=13$.

The (mod p) arithmetic can be accomplished by using the binary form of n, namely 1010101101. Referring to FIG. **7**, the shifts and adds are accomplished by first latching the q (4) least significant bits (LSB's) **701** of n, namely 1101. The remaining bits **702** (101010) are multiplied by C (3) to generate product **703** (1111110). Product **703** is added to bits **701** to generate sum **704** (10001011). The q least significant bits **705** (1011) of sum **704** are latched. The remaining bits **706** (1000) are multiplied by C to generate product **707** (11000). Product **707** is added to bits **705** to generate sum

708 (100011). The q least significant bits 709 (0011) of sum 708 are latched. The remaining bits 710 (10) are multiplied by C to generate product 711 (110). Product 711 is added to bits 709 to generate sum 712 (1001). Sum 712, having q or fewer bits, is the solution to n (mod p). $1001=2^3+2^0=8+1=9$. 685 divided by 13 results in a remainder of 9. The fast class arithmetic provides the solution using only shifts, adds, and multiplies.

Shift and Add Implementation

Fast Mersenne mod operations can be effected via a well known shift procedure. For $p=2^q-1$ we can use:

$$x=(x\&p)+(x>>q) \qquad \text{Equation (17)}$$

a few times in order to reduce a positive x to the appropriate residue value in the interval 0 through p−1 inclusive. This procedure involves shifts and add operations only. Alternatively, we can represent any number x (mod p) by:

$$x=a+b2(q+1)/2=(a, b) \qquad \text{Equation (18)}$$

If another integer y be represented as (c, d), we have:

$$xy \text{ (mod } p)=(ac+2bd,\ ad+bc) \qquad \text{Equation (19)}$$

after which some trivial shift-add operations may be required to produce the correct reduced residue of xy.

To compute an inverse (mod p), there are at least two ways to proceed. One is to use a binary form of the classical extended-GCD procedure. Another is to use a relational reduction scheme. The relational scheme works as follows:

Given $p=2^q-1$, $x\neq0$ (mod p), to return x^{-1} (mod p):

1) Set (a, b)=(1, 0) and (y, z)=(x, p);
2) If (y==0) return(z);
3) Find e such that $2^e//y$;
4) Set $a=2^{q-e}a$ (mod p);
5) If(y==1) return(a);
6) Set (a, b)=(a+b, a−b) and (y, z)=(y+z, y−z);
7) Go to (2).

The binary extended-GCD procedure can be performed without explicit division via the operation $[a/b]_2$, defined as the greatest power of 2 not exceeding a/b:

Given p, and $x\neq0$ (mod p), to return x^{-1} (mod p):

1) If (x==1) return(1);
2) Set (x, v0)=(0, 1) and $(u_1, v_1)=(p, x)$;
3) Set $u_0=[u_1/v_1]2$;
4) Set $(x, v0)=(v_0, x-u_0v_0)$ and $(u_1, v_1)=(v_1, u_1-u_0v_1)$;
5) If $(v_1==0)$ return(x); else go to (3).

The present invention may be implemented on any conventional or general purpose computer system. An example of one embodiment of a computer system for implementing this invention is illustrated in FIG. 4. A keyboard 410 and mouse 411 are coupled to a bi-directional system bus 419. The keyboard and mouse are for introducing user input to the computer system and communicating that user input to CPU 413. The computer system of FIG. 4 also includes a video memory 414, main memory 415 and mass storage 412, all coupled to bi-directional system bus 419 along with keyboard 410, mouse 411 and CPU 413. The mass storage 412 may include both fixed and removable media, such as magnetic, optical or magnetic optical storage systems or any other available mass storage technology. The mass storage may be shared on a network, or it may be dedicated mass storage. Bus 419 may contain, for example, 32 address lines for addressing video memory 414 or main memory 415. The system bus 419 also includes, for example, a 32-bit data bus for transferring data between and among the components, such as CPU 413, main memory 415, video memory 414 and mass storage 412. Alternatively, multiplex data/address lines may be used instead of separate data and address lines.

In the preferred embodiment of this invention, the CPU 413 is a 32-bit microprocessor manufactured by Motorola, such as the 68030 or 68040. However, any other suitable microprocessor or microcomputer may be utilized. The Motorola microprocessor and its instruction set, bus structure and control lines are described in MC68030 User's Manual, and MC68040 User's Manual, published by Motorola Inc. of Phoenix, Ariz.

Main memory 415 is comprised of dynamic random access memory (DRAM) and in the preferred embodiment of this invention, comprises 8 megabytes of memory. More or less memory may be used without departing from the scope of this invention. Video memory 414 is a dual-ported video random access memory, and this invention consists, for example, of 256 kbytes of memory. However, more or less video memory may be provided as well.

One port of the video memory 414 is coupled to video multiplexer and shifter 416, which in turn is coupled to video amplifier 417. The video amplifier 417 is used to drive the cathode ray tube (CRT) raster monitor 418. Video multiplexing shifter circuitry 416 and video amplifier 417 are well known in the art and may be implemented by any suitable means. This circuitry converts pixel data stored in video memory 414 to a raster signal suitable for use by monitor 418. Monitor 418 is a type of monitor suitable for displaying graphic images, and in the preferred embodiment of this invention, has a resolution of approximately 1020×832. Other resolution monitors may be utilized in this invention.

The computer system described above is for purposes of example only. The present invention may be implemented in any type of computer system or programming or processing environment.

Block Diagram

FIG. 8 is a block diagram of the present invention. A sender, represented by the components within dashed line 801, encrypts a plaintext message Ptxt to a ciphertext message C. This message C is sent to a receiver, represented by the components within dashed line 802. The receiver 802 decrypts the ciphertext message C to recover the plaintext message Ptxt.

The sender 801 comprises an encryption/decryption means 803, an elliptic multiplier 805, and a private key source 807. The encryption/decryption means 803 is coupled to the elliptic multiplier 805 through line 809. The elliptic multiplier 805 is coupled to the private key source 807 through line 811.

The encryption/decryption means 804 of receiver 802 is coupled to elliptic multiplier 806 through line. 810. The elliptic multiplier 806 is coupled to the private key source 808 through line 812.

The private key source 807 of the sender 801 contains the secure private password of the sender, "ourPri". Private key source 807 may be a storage register in a computer system, a password supplied by the sender to the cryptosystem when a message is sent, or even a coded, physical key that is read by the cryptosystem of FIG. 8 when a message is sent or received. Similarly, the private key source 808 of receiver 802 contains the secure private password of the receiver, namely, "theirPri".

A separate source 813 stores publicly known information, such as the public keys "ourPub" and "theirPub" of sender 801 and receiver 802, the initial point (x_1, y_1), the field F_{p^k}, and curve parameter "a". This source of information may be a published directory, an on-line source for use by computer

systems, or it may transmitted between sender and receiver over a non-secure transmission medium. The public source **813** is shown symbolically connected to sender **801** through line **815** and to receiver **802** through line **814**.

In operation, the sender and receiver generate a common one time pad for use as an enciphering and deciphering key in a secure transmission. The private key of the sender, ourPri, is provided to the elliptic multiplier **805**, along with the sender's public key, theirPub. The elliptic multiplier **805** computes an enciphering key e_K from (ourPri)°(theirPub) (mod p). The enciphering key is provided to the encryption/decryption means **803**, along with the plaintext message Ptxt. The enciphering key is used with an encrypting scheme, such as the DES scheme or the elliptic curve scheme of the present invention, to generate a ciphertext message C. The ciphertext message is transmitted to the receiver **802** over a nonsecure channel **816**.

The receiver **802** generates a deciphering key D_K using the receiver's private key, theirPri. TheirPri is provided from the private key source **808** to the elliptic multiplier **804**, along with sender's public key, ourPub, (from the public source **813**). Deciphering key D_K is generated from (theirPri)°(ourPub) (mod p). The deciphering key D_K is equal to the enciphering key e_K due to the abelian nature of the elliptic multiplication function. Therefore, the receiver **802** reverses the encryption scheme, using the deciphering key D_K, to recover the plaintext message Ptxt from the ciphertext message C.

The encryption/decryption means and elliptic multiplier of the sender **801** and receiver **802** can be implemented as program steps to be executed on a microprocessor.

Inversionless Parameterization

The use of fast class numbers eliminates division operations in (mod p) arithmetic operations. However, as illustrated by equations 13–16 above, the elliptic multiply operation "°" requires a number of division operations to be performed. The present invention reduces the number of divisions required for elliptic multiply operations by selecting the initial parameterization to be inversionless. This is accomplished by selecting the initial point so that the "Y" terms are not needed.

In the present invention, both sender and recipient generate a mutual one-time pad, as a particular x-coordinate on the elliptic curve. By choosing the initial point (X_1, Y_1) appropriately, divisions in the process of establishing multiples $n°(X_1, Y_1)$ are eliminated. In the steps that follow, the form

$$n°(X_m/Z_m) \qquad \text{Equation (20)}$$

for integers n, denotes the coordinate (X_{n+m}/Z_{n+m}). For x=X/Z the x-coordinate of the multiple n(x, y) as X_n/Z_n, is calculated using a "binary ladder" method in accordance with the adding-doubling rules, which involve multiply mod operations:

If i≠j:

$$X_{i+j}=Z_{i-j}(X_iX_j-Z_iZ_j)^2 \qquad \text{Equation (21)}$$

$$Z_{i+j}=X_{i-j}(X_iZ_j-Z_iX_j)^2 \qquad \text{Equation (22)}$$

Otherwise, if i=j:

$$X_{2i}=(X_i^2-Z_i^2)^2 \qquad \text{Equation (23)}$$

$$Z_{2i}=4X_iZ_i(X_i^2+a\,X_iZ_i+Z_i^2) \qquad \text{Equation (24)}$$

These equations do not require divisions, simplifying the calculations when the present invention is implemented in the present preferred embodiment. This is referred to as "Montgomery parameterization" or "inversionless parameterization" (due to the absence of division operations), and is described in "*Speeding the Pollard and Elliptic Curve Methods of Factorization*" Montgomery, P. 1987 Math. Comp., 48 (243–264). When the field is simply F_p this scheme enables us to compute multiples nx via multiplication, addition, and (rapid) Mersenne mod operations. This also holds when the field is F_p2. Because p=3 (mod 4) for any Mersenne prime p, we may represent any X_i or Z_i as a complex integer, proceeding with complex arithmetic for which both real and imaginary post-multiply components can be reduced rapidly (mod p). We also choose $Z_1=1$, so that the initial point on the curve is $(X_1/1, y)$ where y will not be needed.

Using both fast class numbers and inversionless parameterization, a public key exchange using the method of the present invention can proceed as follows. In the following example, the prime is a Mersenne prime. However, any of the fast class numbers described herein may be substituted.

1) At "our" end, use parameter a, to compute a public key: ourPub ϵ F_{pk}
   (X/Z)=ourPri°$(X_1/1)$
   ourPub=XZ^{-1}
2) At "their" end, use parameter a, to compute a public key: theirPub ϵ F_{pk}
   (X/Z)=theirPri°$(X_1/1)$
   theirPub=XZ^{-1}
3) The two public keys ourPub and theirPub are published, and therefore are known.
4) Compute a one-time pad: ourPad ϵ F_{pk}
   (X/Z)=ourPri°(theirPub/1)
   ourPad=XZ^{-1}
5) Compute a one-time pad: theirPad ϵ F_{pk}
   (X/Z)=theirPri°(ourPub/1)
   theirPad=XZ^{-1}

The usual key exchange has been completed, with ourPad=theirPad

Message encryption/decryption between "our" end and "their" end may proceed according to this mutual pad.

FFT Multiply

For very large exponents, such as q>5000, it is advantageous to perform multiplication by taking Fourier transforms of streams of digits. FFT multiply works accurately, for example on a 68040-based NeXTstation, for general operations xy (mod p) where $p=2^q-1$ has no more than $q=2^{20}$ (about one million) bits. Furthermore, for Mersenne p there are further savings when one observes that order-q cyclic convolution of binary bits is equivalent to multiplication (mod 2^q-1). The use of FFT multiply techniques results in the ability to perform multiply-mod in a time roughly proportional to q log q, rather than q^2.

Elliptic curve algebra can be sped up intrinsically with FFT techniques. Let X denote generally the Fourier transform of the digits of X, this transform being the same one used in FFT multiplication. Then we can compute coordinates from equations 21–24. To compute X_{i+j} for example, we can use five appropriate transforms, (X_i, X_j, Z_i, Z_j, and Z_{i-j}) (some of which can have been stored previously) to create the transform:

$$X_{i+j}=Z_{i-j}(X_iX_j-Z_iZ_j)^2$$

In this way the answer X_{i+j} can be obtained via 7 FFT's. (Note that the usual practice of using 2 FFT's for squaring and 3 FFT's for multiplication results in 11 FFT's for the "standard" FFT approach). The ratio 7/11 indicates a sig-

nificant savings for the intrinsic method. In certain cases, such as when p is a Mersenne prime and one also has an errorless number-theoretic transform available, one can save spectra from the past and stay in spectral space for the duration of long calculations; in this way reducing times even further.

A flow diagram illustrating the operation of the present invention when using fast class numbers, inversionless parameterization and FFT multiply operations is illustrated in FIG. **9**. At step **901**, a fast class number p is chosen where $p=2^q-C$. The term q is the bit depth of the encryption scheme. The greater the number of bits, the greater the security. For large values of q, FFT multiply operations are used to calculate p. The term p is made publicly available.

At step **902**, the element k for the field F_{pk} is chosen and made public. At step **903**, an initial point (X_1/Z) on the elliptic curve is selected. By selecting the initial point to be inversionless, costly divides are avoided. The initial point is made public. The curve parameter a is chosen at step **904** and made public.

At step **905**, the sender computes X_1/Z=ourPri°$(X_1/1)$ using inversionless parameterization. The sender's public key is generated ourPub=(XZ^{-1})(mod p). The receiver's public key theirPub=(XZ^{-1})(mod p), is generated at step **906**.

A one time pad for the sender, ourPad, is generated at step **907**. X/Z=(ourPri)°(theirPub/1). ourPad=XZ^{-1}(mod p). At step **908**, a one time pad for the receiver, theirPad, is generated. X/Z=(theirPri)°(ourPub/1). theirPad=XZ^{-1}(mod p). The calculation of ourPad and theirPad utilizes FFT multiplies to eliminate the need to calculate the inversion Z^{-1}. At step **909**, the sender converts a plaintext message Ptxt to a ciphertext message C using ourPad. The ciphertext message C is transmitted to the receiver. At step **910**, the receiver recovers the plaintext message Ptxt by deciphering the ciphertext message C using theirPad.

FEE Security

The algebraic factor $M_{89}=2^{89}-1$, which is a Mersenne prime, occurs with "natural" statistics when the elliptic curve method (ECM) was employed. This was shown in attempts to complete the factorization of $M_{445}=2^{445}-1$ (this entry in the Cunningham Table remains unresolved as of this writing). In other words, for random parameters a the occurrence $k(X_1/1)=O$ for elliptic curves over F_p with $p=M_{89}$ was statistically consistent with the asymptotic estimate that the time to find the factor M_{89} of M_{445} be O(exp($\sqrt{}$(2 log p log log p)). These observations in turn suggested that finding the group order over F_p is not "accidentally" easier for Mersenne primes p, given the assumption of random a parameters.

Secondly, to check that the discrete logarithm problem attendant to FEE is not accidentally trivial, it can be verified, for particular a parameters, that for some bounded set of integers N

$$(p^N-1)\,(X_1/1)\neq O$$

The inequality avoids the trivial reduction of the discrete logarithm evaluation to the equivalent evaluation over a corresponding finite field. Failures of the inequality are extremely rare, in fact no non-trivial instances are known at this time for q>89.

The present invention provides a number of advantages over prior art schemes, particularly factoring schemes such as the RSA scheme. The present invention can provide the same security with fewer bits, increasing speed of operation. Alternatively, for the same number of bits, the system of the present invention provides greater security.

Another advantage of the present cryptosystem over prior art cryptosystems is the distribution of private keys. In prior art schemes such as RSA, large prime numbers must be generated to create private keys. The present invention does not require that the private key be a prime number. Therefore, users can generate their own private keys, so long as a public key is generated and published using correct and publicly available parameters p, F_{pk}, (X_1/Z) and "a". A user cannot generate its own private key in the RSA system.

DIGITAL SIGNATURE

The present invention provides an improved method for creating and authenticating a digital signature that uses the elliptic algebra described above and a hashing or digesting function. The sender has prepared an encrypted message "ciphertext". This message may be encrypted as described above or may be encrypted using any other encryption scheme. The sender then creates a digital signature to append to the message as a way of "signing" the message. The signature scheme of the preferred embodiment is described below, followed by the method of reducing computations.

Creation of Signature

Assume a curve parameterized by a, with starting point $(X_1/1)$. The sender's public key ourPub is generated as the multiple ourPri°$(x_1/1)$, where ourPri is our private key (an integer) and ° is multiplication on the elliptic curve. The digital signature is created as follows:

1) Choose a random integer m of approximately q bits.
2) Compute the point

$$P=m°(X_1/1).$$

3) Using a message digest function M, compute the integer

$$u=m+\text{our Pri}*M(\text{ciphertext}, P)$$

where ciphertext is the encrypted message to be sent.

4) Along with the ciphertext, transmit the digital signature as the pair (u, P). Note that u is an integer of about 2^q bits, while P is a point on the curve.

In the preferred embodiment of the present invention, a message digesting function M such as MD2 or MD5 is used as part of the creation of the digital signature. However, the present invention may be implemented using other digesting functions or by using any suitable hashing function.

AuthenticatiOn of Digital Signature

The receiver attempts to authenticate the signature by generating a pair of points to match the digital signature pair, using the ciphertext message and the public key of the purported sender. The receiver verifies the signature using the following steps:

1) Using the u part of the signature, compute the point

$$Q=u°(X_1/1)$$

2) Compare the point Q to the point

$$R=P+M(\text{ciphertext}, P)°\text{ourPub}$$

The signature is invalid if these elliptic points Q and R do not compare exactly. In other words, if the signature is authentic, the following must hold:

$$u°(X_1/1)=P+M(\text{ciphertext}, P)°\text{ourPub}$$

Substituting for u on the left side of the equation above gives:

$$(m+\text{our Pri}*M(\text{ciphertext}, P))°(X_1/1)=P+M(\text{ciphertext}, P)°\text{ourPub}$$

or:

$$m°(X_1/1)+(\text{ourPri}*M(\text{ciphertext}, P))°(X_1/1)=P+M(\text{ciphertext}, P)°\text{ourPub}$$

Substituting for ourPub on the right side of the equation yields:

$$m°(X_1/1)+(\text{ourPri}*M(\text{ciphertext}, P))°(X_1/1)=P+M(\text{ciphertext}, P)°\text{ourPri}°(X_1/1)$$

Since $P=m°(X_1/1)$ from above, the left side becomes:

$$P+(\text{ourPri}*M(\text{ciphertext}, P))°(X_1/1)=P+M(\text{ciphertext}, P)°\text{ourPri}°(X_1/1)$$

Moving ourPri in the right side of the equation gives:

$$P+\text{ourPri}*M(\text{ciphertext}, P))°(X_1/1)=P+\text{ourPri}*M(\text{ciphertext}, P)°(X_1/1)$$

Thus, a point on a curve is calculated via two different equations using the transmitted pair (u, P). It can be seen that by calculating Q from the transmitted point u, and by calculating R from transmitted point P, the ciphertext message, and the public key of the purported sender, the digital signature is assumed authenticated when Q and R match.

Security

The digital signature scheme of this scheme is secure on the basis of the following observation. To forge a signature one would need to find a pair (u, P) and a ciphertext that satisfy the equation

$$u°(X_1/1)=P+M(\text{ciphertext}, P)°\text{ourPub}$$

This would either entail an elliptic logarithm operation (the basis of the encryption security of the present invention) or breaking of the hash function M.

Optimizing Authentication

The recipient's final step in the digital signature scheme of the present invention involves the addition of two points; namely P and M(ciphertext, P)°ourPub to yield R and comparing that sum to a point Q. One could perform the elliptic addition using specified y-coordinates at each step. The scheme of the present invention provides a method of deducing the possible values of the x-coordinate of a sum of two points, using only the respective x-coordinates of the original two points in question. Using this method one may rapidly perform a necessity check on whether the points Q and the sum of P+M(ciphertext, P)°ourPub have identical x-coordinates.

A principle for fast verification of sums, using only x-coordinates, runs as follows. Let the curve be

$$By^2=x^3+Ax^2+x$$

Theorem: Let $P_1=(x_1, y_1)$, $P_2=((x_2, y_2)$, and $Q=(x, y)$ be three points on a given curve, with $x_1 \neq x_2$. Then

$$P_1+P_2=Q$$

only if

$$x(c-x)=b^2$$

where

$$b=(x_1x_2-1)/(x_1-x_2)$$

$$C=2\{(x_1x_2+1)(x_1+x_2+2A)-[2A]/(x_1-x_2)^2$$

The proof is given as follows. Not knowing the y-coordinates of P_1 and P_2, the only possibilities for the x-coordinate of the sum P_1+P_2 are, for any fixed pair (y_1, t_2), the respective x-coordinates (call them e,f)) of the two forms $(x_1,y_1)+(x_2, y_2)$. One can compute:

$$ef=b^2$$

$$e+f=c$$

as in Montgomery, supra. Since x is one or the other of e,f it is necessary that $(x-e)(x-f)=0$, whence the quadratic equation of the theorem holds.

Therefore, the quadratic equation $(x-e)(x-f)=0$ will generally have two solutions. One solution corresponds to an authentic signature. The other solution is extremely unlikely to have been selected at random, because the pool of x coordinates is of a size comparable to the elliptic curve. Therefore, when $(x-e)(x-f)=0$ is satisfied, it can be safely assumed that the signature is authentic.

In practical application, P_1 represents the calculated point P that is sent as part of the signature by the sender. P_2 represents the expression M(ciphertext, P)°ourPub. Q of course represents $u°(X_1/1)$. P_1+P_2 represents R and is compared to Q.

Flow Diagrams

FIG. **10** is a flow diagram illustrating the generation of a digital signature using the present invention. At step **1001**, the sender chooses a random integer m. This random integer can be generated using a suitable random number generator for use with a microprocessor. At step **1002** a point P is calculated using m. As noted above, this point is generated using the relation $P=m°(X_1/1)$. in the preferred embodiment of the present invention. However, other schemes may be used for generating point P without departing from the scope of the present invention.

At step **1003**, a second point, u, is calculated using m, P, ourPri, and the ciphertext message. In the preferred embodiment of the invention, this is generated using the relationship u=m+our Pri * M(ciphertext, P). As noted above, hashing functions other than digesting functions MD2 and MD5 can be used. In addition, other relationships can be used to calculate u. It is recommended that if other relationships are used, that m, P, ourPri and the ciphertext message be used. At step **1004**, the calculated pair (u, P) is sent as a digital signature.

FIG. **11** is a flow diagram illustrating the authentication of a digital signature in the present invention. At step **1101** the recipient of the message recieves the digital signature (u, P) and the ciphertext message. At step **1102** the point Q is generated using the point u. In the preferred embodiment, the relationship $Q=u°(X_1/1)$ is used to generate Q. Other relationships may be used depending on what relationships were used to calculate u, P by the sender.

At step **1103** a point P_2 is generated using ourPub and the ciphertext message. In the preferred embodiment, the relationship M(ciphertext, P)°ourPub is used to generate P_2. Other relationships may be used depending on what relationships were used to calculate u, P by the sender.

At step **1104** the x values of P_1 and P_2 are used to determine values b and c and ultimately, e and f. This leads to to possible x values for the sum of P_1 and P_2. At decision block **1105** the argument "e,f=x?" is made to determine if either of the possible x values satisfies the equality of $P_1+P_2=Q$. If neither of the calculated x values satisfy the equation, that is, if the argument at decision block **1105** is

false, the signature is not authentic and is indicated at block **1106**. If one of the x values does satisfy the equation, that is, if the argument at decision block **1105** is true, a valid signature is assumed and indicated at block **1107**.

Block Diagram

FIG. **12** illustrates a block diagram for implementing the digital signature scheme of the present invention. Where elements of FIG. **12** are in common with elements of FIG. **8**, the same element numbers are used. The signature scheme is shown in use with an encryption scheme that uses elliptic multiplication, but this is by way of example only. The present invention can be used with any type of encryptions scheme.

A sender, represented by the components within dashed line **1201**, encrypts a plaintext message Ptxt to a ciphertext message C and generates a signature (u, P). This message C and signature (u, P) is sent to a receiver, represented by the components within dashed line **1202**. The receiver **1202** decrypts the ciphertext message C to recover the plaintext message, and authenticates the signature (u, P).

The sender **1201** comprises an encryption/decryption means **1203**, an elliptic multiplier **805**, a random number generator **1205**, a hasher **1207**, and a private key source **807**. The encryption/decryption means **1203** is coupled to the elliptic multiplier **805** through line **809**. The elliptic multiplier **805** is coupled to the private key source **807** through line **811**. The random number generator **1205** provides random number m on line **1209** to elliptic multiplier **805** and to hasher **1207**. Elliptic multiplier **805** provides point u to the nonsecure channel **816** via line **1211**. The encrypted ciphertext C is provided to hasher **1207** via line **1213**. Hasher **1207** provides point P to nonsecure channel **816** via line **1215**.

The encryption/decryption means **1204** of receiver **1202** is coupled to elliptic multiplier **806** through line **810**. The elliptic multiplier **806** is coupled to the private key source **808** through line **812**. The point u is provided to the elliptic multiplier **806** from the nonsecure channel **816** via line **1212**. Elliptic multiplier **806** generates point Q and provides it to comparator **1208** via line **1216**. Hasher **1206** recieves the ciphertext message C and point P from nonsecure channel **816** via line **1210**, and ourPub from source **813** via line **1218**. Hasher **1206** outputs point R to comparator **1208** via line **1214**.

The private key source **807** of the sender **801** contains the secure private password of the sender, "ourPri". Private key source **807** may be a storage register in a computer system, a password supplied by the sender to the cryptosystem when a message is sent, or even a coded, physical key that is read by the cryptosystem of FIG. **12** when a message is sent or received. Similarly, the private key source **808** of receiver **802** contains the secure private password of the receiver, namely, "theirPri".

A separate source **813** stores publicly known information, such as the public keys "ourPub"and "theirPub" of sender **1201** and receiver **1202**, the initial point (x_1, y_1), the field F_{p^k}, and curve parameter "a". This source of information may be a published directory, an on-line source for use by computer systems, or it may transmitted between sender and receiver over a non-secure transmission medium. The public source **813** is shown symbolically connected to sender **1201** through line **815** and to receiver **1202** and hasher **1206** through lines **814** and **1218** respectively.

In operation, the sender and receiver generate a common one time pad for use as an enciphering and deciphering key in a secure transmission, as described above. The enciphering key is provided to the encryption/decryption means **1203**, along with the plaintext message. The enciphering key is used with an encrypting scheme, such as the DES scheme or the elliptic curve scheme of the present invention, to generate a ciphertext message C. The random number generator **1205** generates random number m and provides it to elliptic multiplier **805**. Elliptic multiplier **805** generates point u and provides it to the receiver via nonsecure channel **816**. The ciphertext message C is provided to the hasher **1207**, along with the random number m and ourPri. Hasher **1207** generates point P and provides it to nonsecure channel **816**. The ciphertext message, along with signature (u, P), is transmitted to the receiver **1202** over a nonsecure channel **816**.

The receiver **1202** generates a deciphering key D_K using the receiver's private key, theirPri. TheirPri is provided from the private key source **808** to the elliptic multiplier **806**, along with sender's public key, ourPub, (from the public source **813**). Deciphering key D_K is generated from (theirPri)°(ourPub) (mod p). The deciphering key D_K is equal to the enciphering key e_K due to the abelian nature of the elliptic multiplication function. Therefore, the receiver **1202** reverses the encryption scheme, using the deciphering key D_K, to recover the plaintext message from the ciphertext message C.

The elliptic multiplier **806** of the receiver **1202** receives point u from the nonsecure channel **816**. The elliptic multiplier **806** generates point Q and provides it to comparator **1208**. Hasher recieves the ciphertext message C and point P from the nonsecure channel **816** and the purported senders public key ourPub from source **813** and generates point R, which it provides to comparator **1208**. Comparator **1208** compares points Q and R and if they match, the signature is assumed to be valid. In the present invention, the comparison of points Q and R is accomplished using the optimized scheme using x values described above.

The encryption/decryption means and elliptic multiplier of the sender **1201** and receiver **1202** can be implemented as program steps to be executed on a microprocessor.

Code

A function to compare signatures using the optimized scheme is as follows:

```
int
signature_compare(key p1, key p2, key p3);
/* Returns non-zero if x(p1) cannot be the x-coordinate of the
sum of two points whose respective x-coordinates are x(p2),
x(p3). */
```

A function to calculate Q and compare it with (P+M(ciphertext, P)°ourPub) is as follows:

```
q = new_public_from_private (NULL, depth, seed);
elliptic_mul (q, u); /* u is the random integer. */
elliptic_mul (our, m); /* m = M(cipertext, P). */
/* Next, use the transmitted point p. */
if(signature_compare (p, our, q))
   fprintf(stderr,"Signature invalid.\n");
```

Encryption/Decryption

The encryption/decryption schemes of the present invention can be implemented in the programming language C. The following are examples of programmatic interfaces (.h files) and test programs (.c files) suitable for implementing the encryption/decryption of the present invention.

```
/* fee.h

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 */

#import "giants.h"

#define DEFAULT_VERSION 1 #define DEFAULT_DEPTH 4 #define DEFAULT_SEED 0
#define MAX_DEPTH 22 #define FEE_TOKEN "scicompg" #define BUF_SIZE 8192
#define KEY_TOO_SHORT 1 #define ILLEGAL_CHARS_IN_KEY 2 #define BAD_TOKEN
3 #define VERSION_PARAM_MISMATCH 4 #define DEPTH_PARAM_MISMATCH 5
#define SEED_PARAM_MISMATCH 6 #define EXP_PARAM_MISMATCH 7 #define
A_PARAM_MISMATCH 8 #define X1_PARAM_MISMATCH 9

typedef giant padkey;

typedef struct {
         int version; int depth; int seed; int exp; int a; int x1;
         padkey x;
} keystruct; typedef keystruct *key;

int hexstr_illegal(char *pub_hex); /* Returns non-zero iff pub_hex is
not a valid hex string. */

void hexstr_to_key(char *str, key public); /* Jams public (assumed pre-
malloced) with hex str contents. */

char * new_hexstr_from_key(key public); /* Mallocs and returns a hex
string representing public. */
```

```
key new_public_from_private(char *private, int depth, int seed); /*
Mallocs and returns a new public key. If private==NULL, depth and seed
are ignored, and the returned key is simply malloc'ed but without
meaningful parameters. If private is a valid string, depth and seed are
used to establish correct elliptic parameters. depth is 0 to MAx_DEPTH
inclusive, while seed = DEFAULT_SEED usually, but may be chosen to be
any integer in order to change the encryption parameters for the given
depth. The depth alone determines the time to generate one-time pads.
 */

char * new_hexstr_from_pad(); /* Malloc's and returns a hex string,
null-terminated, representing the one-time pad. This function is usually
called after a make_one_time_pad() call.
 */

void generate_byte_pad(char *byte_pad, int len); /* Jams byte_pad with
len bytes of the one-time pad. There is no null termination; just len
bytes are modified.
 */

int make_one_time_pad(char *private, key public); /* Calculate the
internal one-time pad. */

void free_key(key pub); /* De-allocate an allocated key. */

void NXWritePublic(NXStream *out, key my_pub); /* Write a key to out
stream. */

void NXReadPublic(NXStream *in, key pub); /* Read a key from in stream.
*/

int keys_inconsistent(key pub1, key pub2); /* Return non-zero if pub1,
pub2 have inconsistent parameters.
 */

int encrypt_stream(NXStream *in, NXStream *out, key their_pub, key
my_pub, char *my_pri); /* Encrypt in to out. If my_pub!=NULL, a
consistency check for equivalent parameters with their_pub is performed,
with possible non-zero error returned (and encryption aborted).
Otherwise, when my_pub==NULL, an internal key is temporarily created for
insertion into the out stream.
 */

int decrypt_stream(NXStream *in, NXStream *out, char *my_pri); /*
Decrypt in to out. Non-zero error value is returned if an internal token
```

```
(that should have been present in the in stream) is not properly
decrypted.
 */

void set_crypt_params(int *depth, int *exp, int *a, int *x1, int *seed);

void str_to_giant(char *str, giant g);

int ishex(char *s);

void byte_to_hex(int b, char *s);

void hex_to_byte(char *s, int *b);

int hexstr_to_int(char **s);

int int_to_hexstr(int n, char *str);

int giant_to_hexstr(giant g, char *str);

void make_base(int exp);

void init_elliptic();

padkey get_pad();

void ell_even(giant x1, giant z1, giant x2, giant z2, int a, int q);

void ell_odd(giant x1, giant z1, giant x2, giant z2, giant xor, giant
zor, int q);

int scompg(int n, giant g);

void elliptic(giant xx, giant zz, giant k, int a, int q);

unsigned char byt(padkey x, int k);

int version_param(key pub);

int depth_param(key pub);

int seed_param(key pub);

int exp_param(key pub);
```

```
int a_param(key pub);

int x1_param(key pub);
```

```
/* keytest.c
   Test program for public key exchange, Usage: > keytest depth
   MyPrivate TheirPrivate

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 */

#import <stdio.h> #import <streams/streams.h> #import "fee.h"

main(int argc, char **argv) {
        key my_pub, their_pub; char *my_pub_str, *their_pub_str; char
        *padstr; int depth;

        if(argc<4) {
                fprintf(stderr, "Usage: keytest depth MyPrivate
                TheirPrivate\n"); exit(0);
        }

        depth = atoi(argv[1]); my_pub =
        new_public_from_private(argv[2], depth, DEFAULT_SEED);
        their_pub = new_public_from_private(argv[3], depth,
        DEFAULT_SEED);

        my_pub_str = new_hexstr_from_key(my_pub); their_pub_str =
        new_hexstr_from_key(their_pub);

        printf("My Public Key:\n%s\n",my_pub_str); printf("Their
        Public Key:\n%s\n",their_pub_str);

        free(my_pub_str); free(their_pub_str);

        make_one_time_pad(argv[2], their_pub); padstr =
        new_hexstr_from_pad(); printf("One-time pad, using My Private
        and Their Public:\n%s\n",padstr); free(padstr);

        make_one_time_pad(argv[3], my_pub); padstr =
        new_hexstr_from_pad(); printf("One-time pad, using Their
        Private and My Public:\n%s\n",padstr); free(padstr);

        free_key(my_pub); free_key(their_pub);

        printf("The two one-time pads should be equivalent.\n");

}
```

```
/* solencrypt.c
   Solitaire encryption for personal files, Usage: > solencrypt <depth>
   file file.ell Private Key:

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 */

#import <stdio.h> #import <streams/streams.h> #import "fee.h"

main(int argc, char **argv) {
          key my_pub; int depth; char *my_pri; NXStream *inStream,
          *outStream;

          if(argc<3) {
fprintf(stderr, "Usage: solencrypt <depth> file file.ell\nPrivate Key:
\nwhere depth is an integer 0 through 22, def ault = 4.\n");
          exit(0); } if(argc==4) depth = atoi(argv[1]); else depth =
          DEFAULT_DEPTH;

/* Next, open the streams. */

          inStream = NXMapFile(argv[argc-2],NX_READONLY); outStream =
          NXOpenMemory(NULL,0,NX_WRITEONLY);

/* Next, get private key, make public key, encrypt stream, blank the
private key in memory. */

          my_pri = (char *) getpass("Private Key: "); my_pub =
          new_public_from_private(my_pri, depth, DEFAULT_SEED);
          encrypt_stream(inStream, outStream, my_pub, my_pub, my_pri);
          bzero(my_pri, strlen(my_pri)); free_key(my_pub);

/* Next, flush and write. */

          NXFlush(inStream); NXFlush(outStream); NXSaveToFile(outStream,
          argv[argc-1]); NXClose(inStream); NXCloseMemory(outStream,
          NX_FREEBUFFER);

}
```

```
/* soldecrypt.c
   Solitaire encryption for personal files, Usage: > soldecrypt file.ell
   file Private Key:

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 */

#import <stdio.h> #import <streams/streams.h> #import "fee.h"

main(int argc, char **argv) {
        char *my_pri; NXStream *inStream, *outStream; int err;

        if(argc<3) {
                fprintf(stderr, "Usage: soldecrypt file.ell
                file\nPrivate Key: \n"); exit(0);
        }

/* Next, open the streams. */

        inStream = NXMapFile(argv[1],NX_READONLY); outStream =
        NXOpenMemory(NULL,0,NX_WRITEONLY);

/* Next, decrypt the stream and blank the private key in memory. */

        my_pri = (char *) getpass("Private Key: "); err =
        decrypt_stream(inStream, outStream, my_pri); bzero(my_pri,
        strlen(my_pri)); if(err) {
                fprintf(stderr,"Error %d: bad private key.\n", err);
                exit(0);
        }

/* Next, write and close. */

        NXSaveToFile(outStream, argv[2]); NXClose(inStream);
        NXCloseMemory(outStream, NX_FREEBUFFER);
}
```

I claim:

1. A method for creating and authenticating a digital signature comprising the steps of:

in a sender computer system;

generating a random integer m;

using m, generating a point P_1 having coordinates (X_1, Y_1);

using m and P_1, generating a point u;

sending the pair (u, P_1) as a digital signature to a receiver computer system;

in said reciever computer system;

using u, generating a point Q having coordinates (X, Y);

using P_1, generating a point P_2 having coordinates (X_2, Y_2);

without using Y_1 and Y_2, testing the equality $P_1+P_2=Q$;

identifying a signature as not authentic when the equality $P_1+P_2=Q$ is not satisfied.

* * * * *

Texts Excerpted From
United States Patent no. 5,581,616
(The Crandall '616 Patent)
Disclosing Data Storage Into a
PUBLICLY ACCESSIBLE REPOSITORY

Column 5, lines 42 through 50

One such scheme is described in Koblitz, supra. The enciphering transformation f_A is used to send a message to user A and f_B is the enciphering transformation used to send a message to user B. User A provides a "signature" P that may include some specific information, such as the time the message was sent or an identification number. User A transmits the signature as $f_B f_A^{-1}(P)$. When user B deciphers the message using f_B^{-1}, the entire message is decoded into plaintext except the signature portion, which remains $f_A^{-1}(P)$. User B then applies user A's public key f_A to obtain P. Since P could only have been encrypted by user A (because only user A knows f_A^{-1}) user B can assume that the message was sent by user A.

EXHIBIT C

Column 5, lines 53 through 65

Assume a public key ourPub generated with a function of a private key ourPri. The signature is generated by first choosing a random integer m of approximately q bits. Next a point $P = m^\circ(X_1/1)$ is computed. A message digest function M is used to compute an integer u that is a function of m, ourPri, and the digested version of the ciphertext message and the computed point P. The computed pair (u, P) is transmitted as the signature.

At the receiving end, the u value of the signature is used to compute the point $Q = u^\circ(X_1/1)$. A point R is calculated using P, the digested version of the ciphertext message and P, and myPub. If R and Q do not compare exactly, the signature is not valid (not genuine).

Column 7, lines 14 through 32

Elliptic Curve Algebra

The elliptic curve used with the present invention is comprised of points $(x,y) \epsilon F_{pk} X F_{pk}$ satisfying:

$$b\ y^2 = x^3 + a\ x^2 + x \qquad \text{Equation (7)}$$

together with a "point at infinity" a.

Sender ("our") and recipient ("their") private keys are assumed to be integers, denoted:

ourPri, theirPri ϵ Z

Next, parameters are established for both sender and recipient. The parameters are: (mod p)

q, so that $p = 2^q - C$ is a fast class number (q is the "bit-depth"). The value q is a publicly known value.

k, so that F_{pk} will be the field, and where k is publicly known.

$(x_1, y_1) \epsilon F_{pk}$, the initial x-coordinate, which is publicly known.

a ϵ F_{pk}, the curve-defining parameter (b is not needed). The value a is also publicly known.

Column 7, line 57 through column 8, line 14

Elliptic Curve Public Key Exchange

It is necessary that both sender and recipient use the same set of such parameters. Both sender and recipient generate a mutual one-time pad, as a particular x-coordinate on the elliptic curve.

In the following description, the terms "our" and "our end" refer to the sender. The terms "their" and "their end" refer to the receiver. This convention is used because the key exchange of the present invention may be accomplished between one or more senders and one or more receivers. Thus, "our" and "our end" and "their" and "their end" refers to one or more senders and receivers, respectively. The public key exchange of the elliptic curve cryptosystem of the present invention is illustrated in the flow diagram of FIG. 3.

Step 301

At our end, a public key is computed: ourPub ϵ F_{pk}

$$ourPub = (ourPri)\circ(x_1, y_1) \qquad \text{Equation (12)}$$

Step 302

At their end, a public key is computed: theirPub ϵ F_{pk}

$$theirPub = (theirPri)\circ(x_1, y_1) \qquad \text{Equation (13)}$$

Step 303

The two public keys ourPub and theirPub are published, and therefore known to all users.

Step 304

A one-time pad is computed at our end: ourPad ϵ F_{pk}

$$ourPad = (ourPri)\circ(theirPub) = (ourPri)\circ(theirPri)\circ(x_1, y_1) \qquad \text{Equation (14)}$$

Step 305

A one-time pad is computed at their end: theirPad ϵ F_{pk}

$$theirPad = (theirPri)\circ(ourPub) = (theirPri)\circ(ourPri)\circ(x_1, y_1) \qquad \text{Equation (15)}$$

The elements $(theirPri)\circ(ourPri)\circ(x_1, y_1)$ being part of a finite field, form an abelian group. Therefore, the order of operation of equations 14 and 15 can be changed without affecting the result of the equations. Therefore:

$$\text{ourPad} = (\text{ourPri})\circ(\text{theirPri})\circ(x_1, y_1) = (\text{theirPri})\circ(\text{ourPri})\circ(x_1, y_1) = \text{theirPad} \quad \text{Equation (16)}$$

Since both the sender and receiver use the same one time pad, the message encrypted by the sender can be decrypted by the recipient, using the one time pad. (Note that step 305 can be executed prior to or contemporaneously with any of steps 301-304).

At step 306, the sender encrypts plaintext message Ptxt using ourPad, and transmits ciphertext message C to the receiver. At step 307, the receiver decrypts ciphertext message C to recover plaintext message Ptxt, using theirPad.

Column 12, line 63 through column 13, line 28

A separate source 813 stores publicly known information, such as the public keys "ourPub" and "theirPub" of sender 801 and receiver 802, the initial point (x_1, y_1), the field F_{pk}, and curve parameter "a". This source of information may be a published directory, an on-line source for use by computer systems, or it may transmitted between sender and receiver over a non-secure transmission medium. The public source 813 is shown symbolically connected to sender 801 through line 815 and to receiver 802 through line 814.

In operation, the sender and receiver generate a common one time pad for use as an enciphering and deciphering key in a secure transmission. The private key of the sender, ourPri, is provided to the elliptic multiplier 805, along with the sender's public key, theirPub. The elliptic multiplier 805 computes an enciphering key e_K from (ourPri)$\circ$(theirPub) (mod p). The enciphering key is provided to the encryption/decryption means 803, along with the plaintext message Ptxt. The enciphering key is used with an encrypting scheme, such as the DES scheme or the elliptic curve scheme of the present invention, to generate a ciphertext message C. The ciphertext message is transmitted to the receiver 802 over a nonsecure channel 816.

The receiver 802 generates a deciphering key D_K using the receiver's private key, theirPri. TheirPri is provided from the private key source 808 to the elliptic multiplier 804, along with sender's public key, ourPub, (from the public source 813). Deciphering key D_K is generated from (theirPri)$\circ$(ourPub) (mod p). The deciphering key D_K is equal to the enciphering key e_K due to the abelian nature of the elliptic multiplication function. Therefore, the receiver 802 reverses the encryption scheme, using the deciphering key D_K, to recover the plaintext message Ptxt from the ciphertext message C.

Column 14, lines 15 through 38

Using both fast class numbers and inversionless parameterization, a public key exchange using the method of the present invention can proceed as follows. In the following example, the prime is a Mersenne prime. However, any of the fast class numbers described herein may be substituted.

1) At "our" end, use parameter a, to compute a public key: ourPub ϵ F_{pk}
   (X/Z) = ourPri° $(X_1/1)$
   ourPub = XZ^{-1}
2) At "their" end, use parameter a, to compute a public key: theirPub ϵ F_{pk}
   (X/Z) = theirPri° $(X_1/1)$
   theirPub = XZ^{-1}
3) The two public keys ourPub and theirPub are published, and therefore are known.
4) Compute a one-time pad: ourPad ϵ F_{pk}
   (X/Z) = ourPri°(theirPub/1)
   ourPad = XZ^{-1}
5) Compute a one-time pad: theirPad ϵ F_{pk}
   (X/Z) = theirPri°(ourPub/1)
   theirPad = XZ^{-1}

The usual key exchange has been completed, with ourPad = theirPad

Column 15, lines 7 through 32

A flow diagram illustrating the operation of the present invention when using fast class numbers, inversionless parameterization and FFT multiply operations is illustrated in FIG. 9. At step 901, a fast class number p is chosen where $p = 2^q - C$. The term q is the bit depth of the encryption scheme. The greater the number of bits, the greater the security. For large values of q, FFT multiply operations are used to calculate p. The term p is made publicly available.

At step 902, the element k for the field F_{pk} is chosen and made public. At step 903, an initial point (X_1/Z) on the elliptic curve is selected. By selecting the initial point to be inversionless, costly divides are avoided. The initial point is made public. The curve parameter a is chosen at step 904 and made public.

At step 905, the sender computes $X_1/Z = ourPri \circ (X_1/1)$ using inversionless parameterization. The sender's public key is generated $ourPub = (XZ^{-1}) (\bmod\ p)$. The receiver's public key $theirPub = (XZ^{-1}) (\bmod\ p)$, is generated at step 906.

A one time pad for the sender, ourPad, is generated at step 907. $X/Z = (ourPri) \circ (theirPub/1)$. $ourPad = XZ^{-1} (\bmod\ p)$. At step 908, a one time pad for the receiver, theirPad, is generated. $X/Z = (theirPri) \circ (ourPub/1)$. $theirPad = XZ^{-1} (\bmod\ p)$. The calculation of ourPad and theirPad utilizes FFT multiplies to eliminate the need to calculate the inversion Z^{-1}.

Column 16, lines 1 through 9

Another advantage of the present cryptosystem over prior art cryptosystems is the distribution of private keys. In prior art schemes such as RSA, large prime numbers must be generated to create private keys. The present invention does not require that the private key be a prime number. Therefore, users can generate their own private keys, so long as a public key is generated and published using correct and publicly available parameters p, F_{pk}, (X_1/Z) and "a". A user cannot generate its own private key in the RSA system.

Column 16, line 48 through column 17, line 28

The receiver attempts to authenticate the signature by generating a pair of points to match the digital signature pair, using the ciphertext message and the public key of the purported sender. The receiver verifies the signature using the following steps:

1) Using the u part of the signature, compute the point

$$Q = u^{\circ}(X_1/1)$$

2) Compare the point Q to the point

$$R = P+M(\text{ciphertext, } P)^{\circ}\text{ourPub}$$

The signature is invalid if these elliptic points Q and R do not compare exactly. In other words, if the signature is authentic, the following must hold:

$$u^{\circ}(X_1/1) = P+M(\text{ciphertext, } P)^{\circ}\text{ourPub}$$

Substituting for u on the left side of the equation above gives:

$$(m+\text{ourPri}*M(\text{ciphertext, } P))^{\circ}(X_1/1) = P+M(\text{ciphertext, } P)^{\circ}\text{ourPub}$$

or:

$$m^{\circ}(X_1/1)+(\text{ourPri}*M(\text{ciphertext, } P))^{\circ}(X_1/1) = P+M(\text{ciphertext, } P)^{\circ}\text{ourPub}$$

Substituting for ourPub on the right side of the equation yields:

$$m^{\circ}(X_1/1)+(\text{ourPri}*M(\text{ciphertext, } P))^{\circ}(X_1/1) = P+M(\text{ciphertext, } P)^{\circ}\text{ourPri}^{\circ}(X_1/1)$$

Since $P = m^{\circ}(X_1/1)$ from above, the left side becomes:

$$P+(\text{ourPri}*M(\text{ciphertext},P))^{\circ}(X_1/1) = P+M(\text{ciphertext, } P)^{\circ}\text{ourPri}^{\circ}(X_1/1)$$

Moving ourPri in the right side of the equation gives:

$$P+\text{ourPri}*M(\text{ciphertext, } P))^{\circ}(X_1/1) = P+\text{ourPri}*M(\text{ciphertext, } P)^{\circ}(X_1/1)$$

Thus, a point on a curve is calculated via two different equations using the transmitted pair (u, P). It can be seen that by calculating Q from the transmitted point u, and by calculating R from transmitted point P, the ciphertext message, and the public key of the purported sender, the digital signature is assumed authenticated when Q and R match.

Column 19, line 54 through column 20, line 37

A separate source 813 stores publicly known information, such as the public keys "ourPub"and "theirPub" of sender 1201 and receiver 1202, the initial point (x_1, y_1), the field F_{pk}, and curve parameter "a". This source of information may be a published directory, an on-line source for use by computer systems, or it may transmitted between sender and receiver over a non-secure transmission medium. The public source 813 is shown symbolically connected to sender 1201 through line 815 and to receiver 1202 and hasher 1206 through lines 814 and 1218 respectively.

In operation, the sender and receiver generate a common one time pad for use as an enciphering and deciphering key in a secure transmission, as described above. The enciphering key is provided to the encryption/decryption means 1203, along with the plaintext message. The enciphering key is used with an encrypting scheme, such as the DES scheme or the elliptic curve scheme of the present invention, to generate a ciphertext message C. The random number generator 1205 generates random number m and provides it to elliptic multiplier 805. Elliptic multiplier 805 generates point u and provides it to the receiver via nonsecure channel 816. The ciphertext message C is provided to the hasher 1207, along with the random number m and ourPri. Hasher 1207 generates point P and provides it to nonsecure channel 816. The ciphertext message, along with signature (u, P), is transmitted to the receiver 1202 over a nonsecure channel 816.

The receiver 1202 generates a deciphering key D_K using the receiver's private key, theirPri. TheirPri is provided from the private key source 808 to the elliptic multiplier 806, along with sender's public key, ourPub, (from the public source 813). Deciphering key D_K is generated from (theirPri)°(ourPub) (mod p). The deciphering key D_K is equal to the enciphering key e_K due to the abelian nature of the elliptic multiplication function. Therefore, the receiver 1202 reverses the encryption scheme, using the deciphering key D_K, to recover the plaintext message from the ciphertext message C.

The elliptic multiplier 806 of the receiver 1202 receives point u from the nonsecure channel 816. The elliptic multiplier 806 generates point Q and provides it to comparator 1208. Hasher recieves the ciphertext message C and point P from the nonsecure channel 816 and the purported senders public key ourPub from source 813 and generates point R, which it provides to comparator 1208. Comparator 1208 compares points Q and R and if

they match, the signature is assumed to be valid. In the present invention, the comparison of points Q and R is accomplished using the optimized scheme using x values described above.